NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2020

MATHEMATICS: PAPER II

MARKING GUIDELINES

Time: 3 hours 150 marks

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SECTION A

QUESTION 1

(a) $m_{QR} = \frac{4-8}{-3-1} = 1$ ✓

$m_{PQ} = \frac{a-4}{2-(-3)} = 1$ ✓

$a - 4 = 5$

$a = 9$ ✓ (3)

(b) Midpoint of ST (1 ; 7) ✓

$m_{ST} = \frac{8-6}{4-(-2)} = \frac{1}{3}$ ✓

Line perpendicular to ST

$y = -3x + c$

$7 = -3(1) + c$ ✓

$c = 10$

$y = -3x + 10$ ✓ (4)

(c) (1) $m_{ED} = \frac{3-1}{7-6} = 2$ ✓

$m_{AB} = 2$ ✓

Therefore ED//AB and gradients are the same ✓ (3)

(2) E (7 ; 3) is the midpoint of CB (proportion theorem ED//AB) ✓

$C(0\,;2)$ ✓

$B(14\,;4)$ ✓✓ (4)

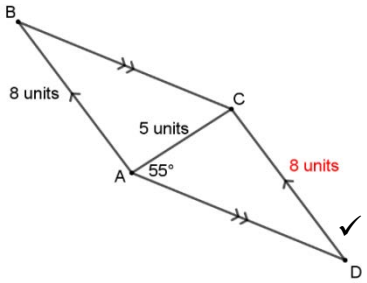
(3) $\tan\theta = 2$ ✓ $\tan\beta = \frac{1}{7}$ ✓

$\theta = 63,43°$ ✓ $\beta = 8,13°$ ✓

$C\hat{B}A = 55,3°$ ✓ (5)

[19]

QUESTION 2



(a) (1) $\frac{\sin \hat{D}}{5} = \frac{\sin 55°}{8}$ ✓✓

$A\hat{D}C = 30,8°$ ✓ ✓ (For the 8 on diagram) (4)

(2) $A\hat{C}D = 94,2°$ ✓

Area of $\Delta ADC = \frac{1}{2}(5)(8)\sin 94,2°$ ✓✓

Area of $\Delta ADC = 19,95$ units^2

Therefore the area of parallelogram ABCD is 39,9 units^2 ✓ (4)

(b) (1) $\cos\theta = \frac{5}{13}$

In quadrant 4:
$x = 5$
$y = -12$ ✓
$r = 13$

$\sin\theta$
$= \frac{-12}{13}$ or $-\frac{12}{13}$ ✓ (2)

(2) $\cos\theta\cos 45° - \sin\theta\sin 45°$ ✓
$= \left(\frac{5}{13}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{-12}{13}\right)\left(\frac{\sqrt{2}}{2}\right)$ ✓✓
$= \frac{17\sqrt{2}}{26}$ (3)

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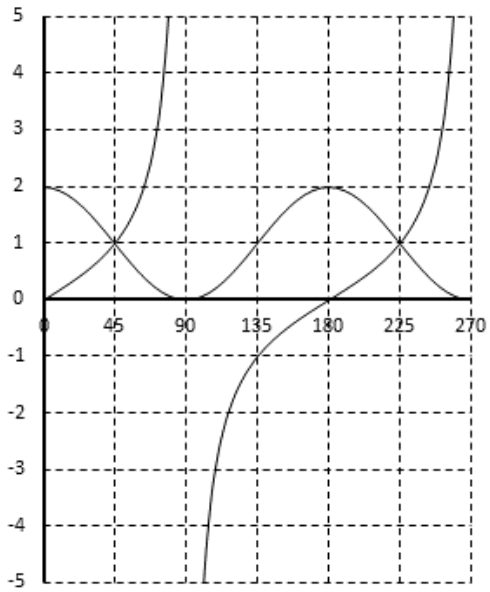
QUESTION 3

(a) Period of graph is 180° ✓ (1)

(b) 90° and 270° ✓✓ (2)

(c)



✓ cos turning points
✓ cos start and end points
✓ cos shape
✓ tan asymptotes
✓ tan start and end points
✓ tan shape through 180°

(6)

(d) $x = 45° + k.180°$ $k \in Z$ ✓✓ (2)

[11]

QUESTION 4

(a) $\tan\theta = 1$ ✓
$\theta = 45° + k.180°$ ✓✓ (3)

(b) (1) $\dfrac{2\sin\theta\cos\theta + 2\cos^2\theta - 1 + 1}{\cos^2\theta - \sin^2\theta}$ ✓✓ $\dfrac{2\cos\theta}{\cos\theta - \sin\theta}$

$\dfrac{2\cos\theta(\sin\theta + \cos\theta)}{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}$ ✓✓

$\dfrac{2\cos\theta}{\cos\theta - \sin\theta}$

Therefore LHS = RHS (4)

(2) $\theta = \{45° ; 135°\}$ ✓✓ (2)

[9]

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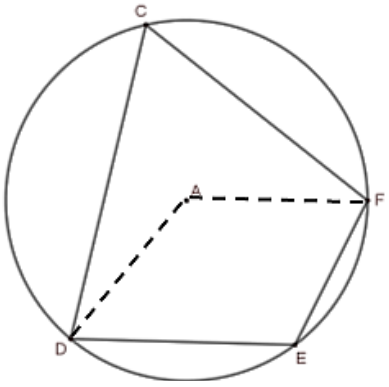
QUESTION 5

(a) Construction: AD and AF ✓ (1)



$\hat{A}_1 = 2\hat{C}$ (Angle at centre = 2 x angle at circumference) ✓
$\hat{A}_2 = 2\hat{E}$ (Angle at centre = 2 x angle at circumference) ✓
$\hat{A}_1 + \hat{A}_2 = 360°$ (Revolution) ✓
$2\hat{C} + 2\hat{E} = 360°$ ✓
therefore
$\hat{C} + \hat{E} = 180°$ (4)

(b) $\hat{E}_1 + \hat{E}_2 = 105°$ ✓ (Opposite angles of a cyclic quad) ✓
$\hat{E}_2 = 73°$ ✓ (Tan chord theorem) ✓
$\hat{E}_1 = 32°$ ✓ (5)

(c) (1) $\hat{D}_2 = 47°$ (Radii, isosceles triangle) ✓ (1)

(2) $\hat{E} = 43°$ (Angle at centre = 2 × angle at circumference) ✓ (1)

(3) $A\hat{F}E = 90°$ ✓ (Line from centre to midpoint of chord) ✓
$\hat{C}_1 = 47°$ (Angles in a triangle) ✓
Therefore DACB is a cyclic quad
(Converse: Exterior angle of a cyclic quad = interior opposite angle) ✓ (4)
[16]

QUESTION 6

(a) (1) $r = 1$ ✓ (1)

(2) Perfect correlation ✓ (1)

(3) $y = -13 + 3x$ ✓✓ (2)

(4) When the x value is way outside of the values used (e.g. 150 or a very small value like 1 or 2). This would be extrapolation. ✓ (1)

(b) (1) 40 people ✓ (1)

(2) 10 out of 140 = 7,1% ✓✓ (2)
[8]

76 marks

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SECTION B

QUESTION 7

(a) (1) IQR = 80 – 60 = 20 ✓
60 – 1,5*20 = 30 ✓
P lies below (60 – 30 = 30) and is therefore an outlier ✓ (3)

(2) Skewed to the left as the mean is to the left of the median ✓✓ (2)

(3) Yes. There were 7 from Class A and 6 from Class B as one of the six is an outlier. ✓✓ (2)

(b) (1) $\frac{(1\times p)+(3\times 165)+(5\times 290)+(7\times 185)+(9\times 75)}{715+p}=5$ ✓✓

$-4p=-340$
$p=85$ ✓ (3)

(2) $Std\ dev=\sqrt{\frac{6\,520}{1\,000}}=2{,}55$ or 2,6 ✓✓ (2)

[12]

QUESTION 8

(a) $9^2=r^2+r^2-2r^2\cos 110°$ ✓✓
$81=2r^2(1-\cos 110°)$
$r=5{,}493\,485\,649$ units ✓
$DB^2=2^2+5{,}493\,485\,649^2$ ✓✓
$DB=5{,}8462$ units ✓ (6)

Alternate solution

$\frac{AB}{\sin 35°}=\frac{9}{\sin 110°}$ ✓✓

$AB = 5{,}493485649$ units ✓
$DB^2 = 2^2+5{,}493485649^2$ ✓✓
$DB = 5{,}8462$ units ✓

(b) $9^2=(5{,}8462)^2+(5{,}8462)^2-2(5{,}8462)^2\cos C\hat{D}B$ ✓✓
$C\hat{D}B=100{,}7°$ ✓ (3)
[9]

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QUESTION 9

(a) $\hat{E}_2 = \hat{B}$ ✓ (Tan chord theorem) ✓

ΔDEC is an isosceles triangle (Tangents drawn from common point) ✓✓

ΔABC is an isosceles triangle (Radii) ✓

Therefore

$\Delta ABC /\!/\!/ \Delta DEC$ (A.A.A) (5)

(b) $\frac{AB}{DE} = \frac{BC}{EC}$ ✓

AB.EC = BC.DE ✓

But AB = AE (Radii) ✓

Therefore AE.EC = BC.DE (3)

[8]

QUESTION 10

(a) (1) $\frac{4}{9}$ ✓✓ (2)

(2) $\frac{HC}{5k} = \frac{3}{8}$ ✓

$HC = \frac{15k}{8}$ ✓

$\frac{HC}{AF} = \frac{\frac{15k}{8}}{4k} = \frac{15}{32}$ ✓✓

(4)

(b) $\hat{A}_1 = 2\hat{B}_1$ ✓ (Angle at centre = 2 x angle on circle) ✓

$\hat{A}_1 = \hat{E}_1 + \hat{B}_1$ ✓ ($\hat{E}_1 = \hat{B}_1$ angles in same segment) ✓

but $\hat{E}_1 = 90° - \hat{E}_2$ ✓ (Angle in semi-circle) ✓

Thus $\hat{A}_1 = 90° - \hat{E}_2 + \hat{B}_1$

Alternate solution

$\hat{E}_2 = 90° - \hat{E}_1$ ✓ (Angles in semi-circle) ✓

$B_1 = \hat{E}_1$ ✓ (Angles in same segment) ✓

$\hat{E}_2 = 90° - \hat{B}_1$

$\hat{A}_2 = 180° - 2\hat{B}_1$

$\hat{B}_1 = 90° - \hat{E}_2$

$\hat{A}_2 = 180° - \hat{B}_1 - \hat{B}_1$

$\hat{A}_2 = 180° - \hat{B}_1 - 90° + \hat{E}_2$ ✓

$\hat{A}_1 = 180° - 90° - \hat{E}_2 + \hat{B}_1$ ✓

$\hat{A}_1 = 90° - \hat{E}_2 + \hat{B}_1$ (6)

[12]

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QUESTION 11

(a) $\hat{C}_1 + \hat{C}_2 + \hat{F}_1 + \hat{F}_2 = 180°$ ✓ (Opposite angles of cyclic quad) ✓

$\hat{B}_1 + \hat{B}_2 + \hat{F}_2 + \hat{F}_3 = 180°$ ✓ (Opposite angles of cyclic quad) ✓

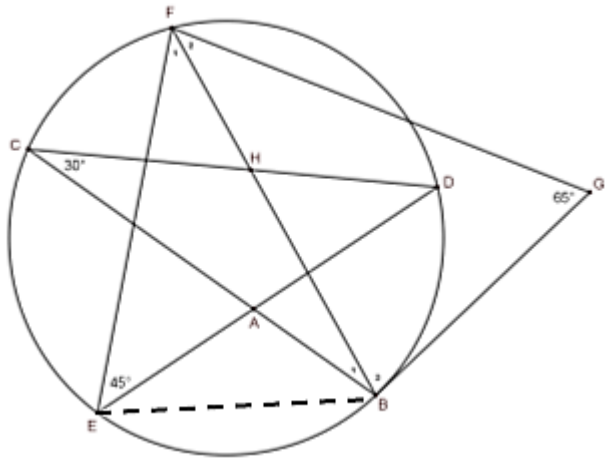
But

$\hat{F}_1 = \hat{F}_3$ ✓ (Angles in same segment) ✓

Therefore

$\hat{C}_1 + \hat{C}_2 = \hat{B}_1 + \hat{B}_2$ (6)

(b)



Construction EB ✓

$B\hat{E}D = 30°$ ✓ (Angles in same segment) ✓

$\hat{B}_2 = 75°$ ✓ (Tan chord theorem) ✓

$\hat{F}_2 = 40°$ ✓ (Angles in a triangle) (6)

[12]

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QUESTION 12

(a) Gradient of radius $\frac{1}{2}$ ✓

Sub in point P(2 ; 2)

$2 = \frac{1}{2}(2) + c$ ✓

$c = 1$ ✓ (3)

(b) $0 = -2x + 6$

$x = 3$

B(3 ; 0) ✓

$m_{BT} = \frac{1}{2}$ ✓

Gradient of line through centre = –2 ✓

$y = -2x + c$

$1 = -2(5) + c$

$c = 11$

$y = -2x + 11$ ✓

$-2x + 11 = \frac{1}{2}x + 1$ ✓

$x = 4$

$y = 3$

Radius of circle

$r = \sqrt{(3-2)^2 + (4-2)^2}$

$r = \sqrt{5}$ ✓

Minimum distance from the *x*-axis

$3 - \sqrt{5}$ OR 0,76 ✓ (7)

[10]

QUESTION 13

(a) $x^2 + 2x + 1 + y^2 - 2y + 1 = 1$

$(x+1)^2 + (y-1)^2 = 1$ ✓

Distance between centres

$d = \sqrt{(4-1)^2 + (3-(-1))^2}$ ✓

$d = \sqrt{25}$

$d = 5$ ✓

Radius 1 + Radius 2

1 + 2 = 3 ✓

Distance between the centres – (Radius 1 + Radius 2) = 2 metres ✓

Therefore

The areas will never intersect (5)

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(b) Equation of quarter circle

$\frac{\pi r^2}{4} = 8\pi$ ✓

Radius of quarter circle $= \sqrt{32}$ ✓

Equation of circle centre A

$(x-2)^2 + (y-2)^2 = 4$

$FA = \sqrt{(-1-2)^2 + (5-2)^2}$

$FA = \sqrt{18}$ ✓

$AB = \sqrt{32} - \sqrt{18}$

$AB = \sqrt{2}$ ✓

$BC^2 = 11 - 2$

$BC = 3$ ✓

Therefore

Perimeter of ABCD $= 6 + 2\sqrt{2}$ ✓

Alternative Solution

Equation of FB

$m_{FB} = -1$

$2 = -1(2) + c$

$c = 4$

Coordinates of B

$y = -x + 4$ ✓

$(x-5)^2 + (-x+4+1)^2 = 32$ ✓✓

$x^2 - 10x + 25 + x^2 - 10x + 25 = 32$

$2x^2 - 20x + 18 = 0$

$x^2 - 10x + 9 = 0$

$(x-9)(x-1) = 0$

$x = 9$ *or* $x = 1$

Coordinates B(1 ; 3)

AB $= \sqrt{2}$ ✓ (Use of the distance formula)

$BC^2 = 11 - 2$

$BC = 3$ ✓

Therefore

Perimeter of ABCD $= 6 + 2\sqrt{2}$ ✓ (6)

[11]

74 marks

Total: 150 marks

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