NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2020

TECHNICAL MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours 150 marks

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QUESTION 1

1.1 1.1.1

$$2x^2 - 5x = 12$$
$$2x^2 - 5x - 12 = 0 \ \checkmark$$
$$(2x+3)(x-4) = 0 \ \checkmark$$
$$x = -\frac{3}{2} \text{ or } x = 4 \ \checkmark$$

(3)

1.1.2

$$4x + 7 = -2x^2$$
$$2x^2 + 4x + 7 = 0$$
$$x^2 + 2x + \frac{7}{2} = 0$$

OR

Option 1

Use formula

$$x = \frac{-4 \pm \sqrt{16 - 56}}{2} \ \checkmark\checkmark$$
$$= \frac{-4 \pm \sqrt{-40}}{2} \ \checkmark$$
$$= \frac{-2 \pm i\sqrt{10}}{2} \ \checkmark \text{ (use calculated)}$$
$$= -1 \pm \frac{\sqrt{10}}{2}i \ \checkmark$$

Option 2

$$(x+1)^2 = 1 - \frac{7}{2} = \frac{-5}{2} = \frac{-10}{4} \ \checkmark\checkmark$$
$$x + 1 = \pm\frac{\sqrt{10i}}{2} \ \checkmark$$
$$x = -1 \pm \frac{\sqrt{10i}}{2} \ \checkmark$$

(4)

1.2 Let energy consumption of motor $= x$ and heater $= y$

$4x + 2y = 25$ ① ✓ and $2x + 3y = 18$ ② ✓

② × 2: $4x + 6y = 36$ ✓

Subtr. $-4y = -11$

$$y = \frac{11}{4} \text{ kJ/hr} \ \checkmark$$

Subst ① $4x + \frac{11}{2} = 25$

$$4x = \frac{39}{2}$$
$$x = \frac{39}{8} \text{ kJ/hr} \ \checkmark$$

(5)

1.3

$$\Delta = (-3)^2 - 4(1)(9k) \ \checkmark$$
$$= 9 - 36k \ \checkmark$$

For real, diff roots $\Delta > 0$

$$9 - 36k > 0 \ \checkmark$$
$$-36k > -9$$
$$k < \frac{1}{4} \ \checkmark$$

(4)

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QUESTION 2

2.1
$$\frac{3^{2014}+9^{1007}}{27^{671}}$$
$$=\frac{3^{2014}+3^{2014}}{3^{2013}}\ \checkmark\checkmark$$
$$=\frac{2.3^{2014}}{3^{2013}}\ \checkmark$$
$$=2.3=6\ \checkmark$$

OR

$$\frac{3^{2014}}{3^{2013}}+\frac{3^{2014}}{2^{2013}}\ \checkmark$$
$$=3+\ \checkmark\ 3=6\ \checkmark$$

(4)

2.2 2.2.1
$$5-\sqrt{4x+1}=x$$
$$5-x=\sqrt{4x+1}\ \checkmark$$
$$25-10x+x^2=4x+1\ \checkmark$$
$$x^2-14x+24=0\ \checkmark$$
$$(x-2)(x-12)=0\ \checkmark$$
$$x=2\ \checkmark \text{ or } x=12 \text{ N/A}\ \}\ \checkmark\checkmark$$

OR Check solution

$5-x\geq 0$ and $4x+1\geq 0$

$-x\geq -5$ and $x\geq -\frac{1}{4}$

$x\leq 5$

(6)

2.2.2
$$2\log x=\log 4+\log(x-1)$$
$$\log x^2=\log(4x-4)\ \checkmark\checkmark\ x>1$$
$$x^2=4x-4$$
$$x^2-4x+4=0\ \checkmark$$
$$(x-2)^2=0\ \checkmark$$
$$x=2\ \checkmark$$

(5)

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QUESTION 3

3.1 $\frac{(3-2i)}{(1+5i)}.\frac{(1-5i)}{(1-5i)}$ ✓

$= \frac{3-17i+10i^2}{1-25i^2}$ ✓

$= \frac{3-17i-10}{1+25}$ $(i^2=-1)$ ✓

$= \frac{-7-17i}{26}$ ✓

$= \frac{-7}{26}-\frac{17}{26}i$ ✓ (5)

3.2 3.2.1 $V = 2(\cos 120° + i\sin 120°)$ ✓ ✓ **or** $2(-\cos 60° + i\sin 60°)$ (2)

3.2.2 $V = 2\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)$ ✓✓

$= -1+\sqrt{3}i$ ✓ ✓ (4)

3.3 11001_2

$= 2^4 + 2^3 + 2^0$ ✓

$= 16 + 8 + 1$ ✓

$= 25$ ✓ (3)

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QUESTION 4

4.1 $= 4\,800 - \left(4\,800 \times \frac{13,5}{100}\right)$ ✓

$= \text{R}4\,152$ ✓

OR $\text{R}4\,800 \times 86,5\%$ ✓

$= \text{R}4\,152$ ✓ (2)

4.2 4.2.1 $m = 4$

$i = 0,09$

$$1 + i = \left(1 + \frac{i^{(m)}}{m}\right)^m$$

$$1 + 0,09 \text{ ✓} = \left(1 + \frac{i^{(4)}}{4}\right)^4 \text{ ✓}$$

$$\sqrt[4]{1,09} \text{ ✓} = 1 + \frac{i^{(4)}}{4}$$

$$\sqrt[4]{1,09} - 1 = \frac{i^{(4)}}{4}$$

$$4\left(\sqrt[4]{1,09 - 1}\right) = i^{(4)} \text{ ✓}$$

$$\therefore i^{(4)} = 8,71\% \text{ ✓}$$

OR The nominal interest rate is 8,71% p.a. compounded quarterly ✓ (5)

4.2.2 Let amount invested $= x$ 2

$$3x = x(1 + 15i) \text{ ✓}$$

$$3 = 1 + 15i \text{ ✓}$$

$$i = \frac{2}{15} = 0,133 \text{ ✓}$$

i.e. $13\frac{1}{3}\%$ p.a. ✓ (4)

4.3 Original value $= x$

$$\frac{x}{2} = x(1 - 0,13)^n \text{ ✓}$$

$$0,5 = 0,87^n \text{ ✓}$$

$$\log_{0,87} 0,5 = n \text{ ✓}$$

$n = 4,977 \ldots$ i.e. 5 years ✓ (4)

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QUESTION 5

5.1 Subst $(-2;5)$ in g: $5 = -(-2) + k$ ✓✓

$3 = k$ (2)

5.2 T is $(0;3)$ i.e. $c = 3$ ✓

$f(x) = -\frac{1}{2}x^2 + bx + 3$ ✓

Subst $(-2;5)$: $5 = -\frac{1}{2}(-2)^2 + b(-2) + 3$ ✓

$2b = -4$

$b = -2$ ✓

OR

$y = -\frac{1}{2}(x+2)^2 + 5$ ✓

$= -\frac{1}{2}x^2 + 2x - 2 + 5$ ✓

$= -\frac{1}{2}x^2 - 2x - 3$ ✓

$c = 3$; $b = -2$ ✓ (4)

5.3 $PQ = y_P - y_Q$

$= (-x + 3) - \left(-\frac{1}{2}x^2 - 2x + 3\right)$ ✓

$= \frac{1}{2}x^2 + x$ ✓ (2)

5.4 $\frac{1}{2}x^2 + x = 12$ ✓

$x^2 + 2x - 24 = 0$

$(x+6)(x-4) = 0$ ✓

$x = -6$ or $x = 4$

P ✓ N/A ✓

Subst in g: $y_P = -(-6) + 3 = 9$ ✓

P is $(-6;9)$ (5)

5.5 $g(x) \leq f(x)$

✓

$-2 \leq x \leq 0$ ✓ **OR** ✓ ✓ $[-2\ ;\ 0]$ (2)

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QUESTION 6

6.1 $y = 1$ ✓ (1)

6.2 At B, $x = 0$: $y = 2^\circ + 1$ ✓

$= 1 + 1 = 2$ ✓

i.e. $r = 2$ so $g(x) = \sqrt{4 - x^2}$ ✓ (3)

6.3 Domain: $x \in [-2\,;2]$ ✓ **OR** $-2 \le x \le 2$

Range: $y \in [0\,;2]$ ✓ $0 \le y \le 2$ (2)

6.4 $M_{AB} = \dfrac{y_A - y_B}{x_A - x_B}$ ✓

$0{,}44 = \dfrac{k-2}{-1{,}466 - 0}$ ✓

$0{,}44(-1{,}466) = k - 2$ ✓

$k \approx 1{,}35$ ✓ (4)

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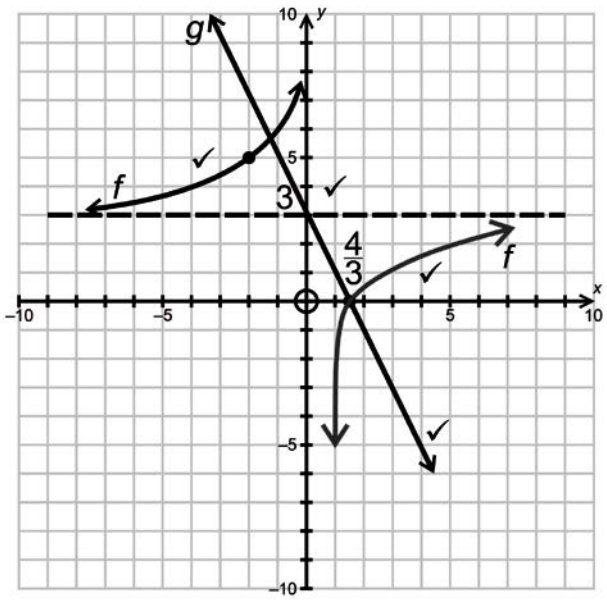
QUESTION 7

7.1



A asymptote $y = 3$

Put $y = 0$ $\quad 0 = -\frac{4}{3} + 3 \quad \Rightarrow x = \frac{4}{3}$

Marks allocated on graph. (4)

7.2 In $g = \frac{3-0}{0-4} = -\frac{9}{4}$ ✓

Eqn of $g : y = -\frac{9}{4}x + 3$ ✓

$$\therefore h(x) = -\frac{9}{4}x + 3 + 1$$

$$= -\frac{9}{4}x + 4 \ \checkmark$$ (3)

[7]

QUESTION 8

8.1
$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \checkmark$$
$$= \lim_{h\to 0} \frac{5-2x-2h-5+2x}{h} \checkmark$$
$$= \lim_{h\to 0} \frac{-2h}{h} \checkmark$$
$$= \lim_{h\to 0} -2 \checkmark$$
$$= -2 \checkmark$$ (5)

8.2
$$y = \frac{x^2}{x} - \frac{4x}{x} + \frac{3}{x} \checkmark$$
$$= x - 4 + 3x^{-1} \checkmark$$
$$\therefore \frac{dy}{dx} = 1 - 3x^{-2} \checkmark\checkmark$$

OR

$$= 1 - \frac{3}{x^2}$$ (4)

8.3
$$f(x) = 2x^{\frac{1}{2}} + x^{-3} - \sqrt{2}x \checkmark$$
$$f'(x) = x^{-\frac{1}{2}} \checkmark - 3x^{-4} \checkmark - \sqrt{2} \checkmark$$

OR

$$= \frac{1}{\sqrt{x}} - \frac{3}{x^4} - \sqrt{2}$$ (4)

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QUESTION 9

9.1 9.1.1 (a)

$$\text{Vol} = \pi r^2 h$$
$$375 = \pi r^2 h \ \checkmark$$
$$\frac{375}{\pi r^2} = h \ \checkmark \qquad (2)$$

(b)

$$\text{SA} = 2\pi r^2 + 2\pi rh$$
$$= 2\pi r^2 + \frac{2\pi r.375}{\pi r^2} \ \checkmark\checkmark$$
$$= 2\pi r^2 + \frac{750}{r} \qquad (2)$$

9.1.2

$$S = 750r^{-1} + 2\pi r^2 \ \checkmark$$
$$\frac{ds}{dr} = -750r^{-2} + 4\pi r \ \checkmark$$
$$\text{At min } \frac{ds}{dr} = 0$$
$$\frac{-750}{r^2} + 4\pi r = 0 \ \checkmark$$
$$-750 + 4\pi r^3 = 0$$
$$r^3 = \frac{750}{4\pi} = \frac{375}{2\pi} \ \checkmark$$
$$r = \sqrt[3]{\frac{375}{2\pi}} \text{ cm } \checkmark \quad \textbf{OR} \quad 5\sqrt[3]{\frac{3}{2\pi}} \qquad (5)$$

9.2 9.2.1

$$f(x) = (x+2)(x-3)^2 \ \checkmark$$
$$= (x+2)(x^2 - 6x + 9) \ \checkmark$$
$$= x^3 - 6x^2 + 9x + 2x^2 + 2x + 18$$
$$= x^3 - 4x^2 - 3x + 18$$
$$p = -4 \ \checkmark \quad q = 18 \ \checkmark$$

OR by substitution

$$0 = (-2)^3 + p(-2)^2 - 3(-2) + q \ \checkmark$$

and

$$0 = (3)^3 + p(3)^2 - 3(3) + q \ \checkmark$$

and solve for p to q $\checkmark$ $\checkmark$ (4)

9.2.2

$$m\tan = f'(x) = 3x^2 - 8x - 3 \ \checkmark$$
$$\text{At A, } 3x^2 - 8x - 3 = 8 \ \checkmark$$
$$3x^2 - 8x - 11 = 0$$
$$(3x - 11)(x + 1) = 0 \ \checkmark$$
$$x = \frac{11}{3} \text{ or } x_A = -1 \ \checkmark$$
$$y_A = (-1+2)(-1-3)^2 \qquad \textbf{OR} \qquad \text{subs in eqn}$$
$$= 16 \ \checkmark \qquad A(-1;16) \qquad (5)$$

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9.2.3 $f(x) = x^3 - 4x^2 - 3x + 18$

$f'(x) = 0$ ✓ (t.p.)

$\therefore 3x^2 - 8x - 3 = 0$

$(3x + 1)(x - 3) = 0$ ✓

$\therefore x = -\frac{1}{3}$ ✓ or $x = 3$

Turning points $\left(-\frac{1}{3}; \frac{500}{27}\right)$ ✓ ; (3 ; 0) ✓ (5)

9.2.4 At K, $m\tan = f'(0) = -3$ ✓

$\therefore m\perp = \frac{1}{3}$ ✓

K is $(0;18)$

$\therefore h(x) = \frac{1}{3}x + 18$ ✓ (3)

9.3 9.3.1 $A = -t^2 + 5t + 8$

$A = -(0)^2 + 5(0) + 8$ ✓

$= 8$ cm^2 ✓ (2)

9.3.2 $\frac{dA}{dt} = -2t + 5$ ✓

at $t = 2$

$= -2(2) + 5$ ✓

$= 1$ cm^2 ✓ (3)

9.3.3 $\frac{dA}{dt} = 0$ ✓

$-2t + 5 = 0$

$t = \frac{5}{2}$ seconds ✓ (2)

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QUESTION 10

10.1 (a) $\int 0dx = c$ ✓ (1)

(b) $\int dx = x + c$ ✓ (1)

10.2 Determine: $\int\left(3x^2 + \frac{1}{x}\right)dx$

$= \frac{3x}{3} + \ln x + c$

$= x^3$ ✓ $\ln x$ ✓ $+ c$ ✓ (3)

10.3 Area $= \int_0^2 x^3 dx$ ✓

$= \left[\frac{x^4}{4}\right]_0^2$ ✓

$= \left[\frac{x^4}{4}\right]_0$ ✓

$= \frac{16}{4} - 0$ ✓

$= 4$ ✓

and by symmetry $\int_{-2}^0 x^3 dx = 4$ ✓

$\therefore$ Total area $= 8$ units^2 ✓

OR

$\int_{-2}^0 x^3 dx = \left[\frac{x^4}{0}\right]_{-2}^0$ ✓

$= -4$

$\therefore$ 8 units^2 ✓ (7)

[12]

Total: 150 marks

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