INTERNASIONALE SEKONDÊRE SERTIFIKAAT-EKSAMEN
NOVEMBER 2022

VERDERE STUDIES WISKUNDE (UITGEBREID): VRAESTEL II

NASIENRIGLYNE

Tyd: 1 uur 100 punte

Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en hulpeksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.

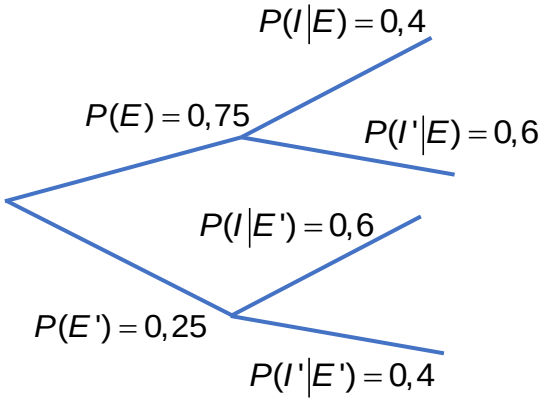
Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.

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MODULE 2 STATISTIEK

VRAAG 1

1.1 (a)



$$P(E|I)=\frac{(0,75)(0,4)}{(0,75)(0,4)+(0,25)(0,6)}=\frac{2}{3}$$

(b)

	Vorige infeksie	Geen vorige infeksie	Totaal
Ingeënt	30%	45%	75%
Nie ingeënt nie	15%	10%	25%
Totaal	45%	55%	100%

$$P(E|I)=\frac{30\%}{45\%}=\frac{2}{3}$$

1.2 (a) $P(X\geq 2)=0,49$

(b) $X\sim B(10;0,49)$

$$P(X>8)=P(X=9)+P(X=10)$$
$$=\binom{10}{9}(0,49)^9(0,51)+\binom{10}{10}(0,49)^{10}(0,51)^0$$
$$=0,0091$$

(c) $Y\sim B(n;0,13)$

$$P(Y\geq 1)>0,7$$
$$1-\binom{n}{0}(0,13)^0(0,87)^n>0,7$$
$$(0,87)^n<0,3$$
$$n>\frac{\log 0,3}{\log 0,87}$$
$$n>8,64$$
$$\therefore n=9$$

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1.3 $X \sim B\left(160;\frac{1}{8}\right) \rightarrow X \sim N\left(20;\sqrt{17,5}^2\right)$

$P(X<25) \rightarrow P(X<24,5)$

$= P\left(Z < \frac{24,5-20}{\sqrt{17,5}}\right)$

$= P(Z<1,08)$

$= 0,5 + 0,3599$

$= 0,8599$

VRAAG 2

2.1 $X \sim N(45;2^2)$

$P(X<41 \text{ of } X>47) = 1-P(41<X<47)$

$= 1-P\left(\frac{41-45}{2} < Z < \frac{47-45}{2}\right)$

$= 1-P(-2<Z<1)$

$= 1-(0,4772+0,3413)$

$= 0,1815$

2.2 $P(X<a) = 0,9$

$1,28 = \frac{a-45}{2}$

$a = 47,56$

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VRAAG 3

3.1 $H_0 : \mu = 27$

$H_1 : \mu \neq 27$

Verwerpingsgebied: verwerp H_0 indien $|Z| > 2{,}58$

Toetsstatistiek: $Z = 2{,}4$

Gevolgtrekking: aangesien $Z < 2{,}58$, versuim om H_0 by die 1%-betekenispeil te verwerp en stel onvoldoende bewys voor om die bewering te ondersteun.

3.2 (a) $$\frac{k}{80} = \frac{0{,}468 + 0{,}682}{2}$$

$$k = 80\left(\frac{23}{40}\right)$$

$$k = 46$$

(b) $$0{,}575 + Z\sqrt{\frac{(0{,}575)(0{,}425)}{80}} = 0{,}682$$

$$Z = 1{,}94$$

$$\therefore P(-1{,}94 < Z < 1{,}94) = 2(0{,}4738)$$

$$= 0{,}9476$$

$$\therefore \alpha = 95\%$$

(c) 59,3% is in die vertrouensinterval, gevolglik is daar bewyse om te sê dat die proporsie mense wat gestem het, nie verander het nie.

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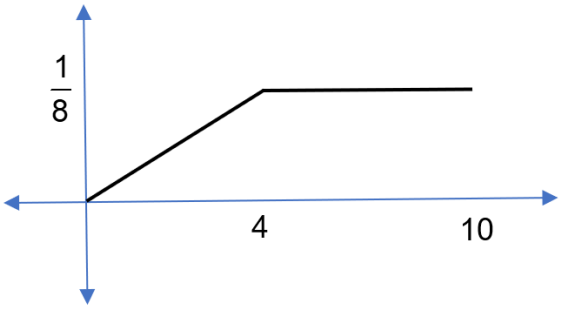
VRAAG 4

4.1 (a) $E[X] = 1(0,3-m)+2(2m)+3(0,7-m)$
$= 0,3-m+4m+2,1-3m$
$= 2,4$

(b) $0,3-m \geq 0$ $\quad 2m \geq 0$ $\quad 0,7-m \geq 0$
$m \leq 0,3$ $\quad m \geq 0$ $\quad m \leq 0,7$
$\therefore 0 \leq m \leq 0,3$

(c) $\text{var}(X) = E[X^2]-(E[x])^2$
$0,72 = E[X^2]-(2,4)^2$
$E[X^2] = 6,48$
$0,3-m+4(2m)+9(0,7-m) = 6,48$
$-2m = -0,12$
$m = 0,06$

4.2 (a)



(b) $P(X>3) = 1-\frac{1}{2}(3)\left(\frac{3}{32}\right) = \frac{55}{64}$

(c) $1-\left(\frac{55}{64}\right)^2 = 0,2615$

OF

$\left(\frac{55}{64}\right)\left(\frac{9}{64}\right)\times 2+\left(\frac{9}{64}\right)^2 = 0,2615$

VRAAG 5

5.1 $(7-1)! \times 2 = 1440$

5.2 Laat $P(X) = P(Y) = P(X' \cap Y') = x$

$$\therefore P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$$
$$1 - P(X' \cap Y') = P(X) + P(Y) - P(X).P(Y)$$
$$1 - x = x + x - x^2$$
$$x^2 - 3x + 1 = 0$$
$$x = 0{,}382 \quad \text{of} \quad x \neq 2{,}618$$

OF

Laat $P(X|Y) = P(X) = P(Y) = P(X' \cap Y') = k$

$$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$$
$$\therefore P(X \cup Y) = 2k - P(X \cap Y)$$
$$P(X' \cap Y') = 1 - P(X \cup Y)$$
$$\therefore k = 1 - 2k + P(X \cap Y)$$
$$P(X|Y) = \frac{P(X \cap Y)}{P(Y)}$$
$$\therefore P(X \cap Y) = k^2$$
$$\therefore k = 1 - 2k + k^2$$
$$k^2 - 3k + 1 = 0$$
$$k = 0{,}382$$

Totaal vir Module 2: 100 punte

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MODULE 3 FINANSIES EN MODELLERING

VRAAG 1

1.1 $$550\,000\left(1+\frac{0{,}08}{12}\right)^{5}=\frac{6000\left[1-\left(1+\frac{0{,}08}{12}\right)^{-n}\right]}{\frac{0{,}08}{12}}$$

$n = 150{,}34$

$\therefore$ 151 betalings

1.2 Uitstaande saldo na 150 betalings =

$$\text{US}(2\,\text{jaar})=550\,000\left(1+\frac{0{,}08}{12}\right)^{155}-\frac{6000\left[\left(1+\frac{0{,}08}{12}\right)^{150}-1\right]}{\frac{0{,}08}{12}}:$$

$$=\text{R}2\,090{,}18$$

$\therefore$ Finale betaling = $2090{,}19\left(1+\frac{0{,}08}{12}\right)$

= R2 104,12

1.3 $$\text{US}(2\,\text{jaar})=550\,000\left(1+\frac{0{,}08}{12}\right)^{24}-\frac{6000\left[\left(1+\frac{0{,}08}{12}\right)^{19}-1\right]}{\frac{0{,}08}{12}}$$

$$=\text{R}523\,982{,}93$$

$$\text{US}(3\,\text{jaar})=550\,000\left(1+\frac{0{,}08}{12}\right)^{36}-\frac{6000\left[\left(1+\frac{0{,}08}{12}\right)^{31}-1\right]}{\frac{0{,}08}{12}}$$

$$=\text{R}492\,773{,}70$$

1.4

$12\times 6000-(523\,982{,}93-492\,773{,}70)$

$=\text{R}40\,790{,}77$

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VRAAG 2

2.1 Bedrag gespaar:

$$\frac{30\,000\left[\left(1+\frac{0,1}{12}\right)^{49}-1\right]\bullet\left(1+\frac{0,1}{12}\right)^{36}}{\frac{0,1}{12}}$$
$$= \text{R}2\,435\,295,52$$

2.2 $1\,750\,000(1+0,06)^{7}-x(1-0,2)^{7}=2\,435\,295,52$ [spasiëring vir miljoene]

2.3 (a) $\left(1+\frac{r}{2}\right)^{2}=\left(1+\frac{0,1}{12}\right)^{12}$

r = 0,1021066 …

(b) $2\,450\,000=\frac{y\left[\left(1+\frac{0,1}{12}\right)^{85}-1\right]}{\frac{0,1}{12}}+\frac{50\,000\left[\left(1+\frac{0,1021}{2}\right)^{14}-1\right]}{\frac{0,1021}{2}}$

A = 2 450 000

q = 50 000

m = 14

n = 85

i = 0,00833

j = 0,05105

(c) $y = \text{R}11\,897,32$

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VRAAG 3

3.1 6 werpsels per 12 maande

$\frac{12}{6}$ = 2 maande

3.2 1/6 of 16,67% (twee desimale aanvaar)

3.3 $r = b - d$

$= 0,55 \times 0,8 \times 4 - 1/6$
$= 1.5933$

3.4 $P_{n+1} = 2,5933Pn$ $P_o = 30$

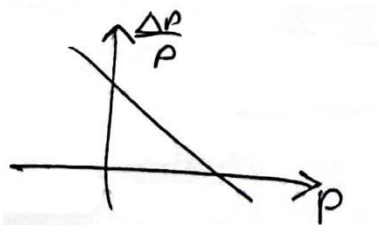
$P_o = 2.5933P_o - x$
$x = 1,5933 \times 30$
= 47,799
$\therefore$ 48 minimum

VRAAG 4

4.1 $x = \frac{496,163 - 32,878}{2 \times 169,075}$
= 1,370

4.2 $\frac{729,835 - 632,054}{2y} = 0,072$
$\therefore y$ = 679,035 (of antwoord kan in 1000s gegee word)

4.3 Toon dat $\frac{\triangle P}{P}$ gestip teen P 'n reguitlyn is met negatiewe gradiënt.



Krediet kan ook gegee word vir s-vorminge kurwe wat infleksie en asimptoot noem.

4.4 O = –0,00339K + 2,335
$\therefore$ K = 689
r = 2,335

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VRAAG 5

5.1 (A) OF (D)

5.2 $f.b.R_n.F_n = 10$ $b = 0,04$

$0,0312 \times 0,04 \times R_O \times 40 = 10$

$\therefore R_O = 200$

(C)

5.3 $F_{n+1} = F_n + fbR_nF_n - cF_n$

$F_{n+1} = F_n + 0,00296R_nF_n - 0,02F_n$

$\text{Leeftyd} = \dfrac{1}{0,02}$

= 50 maande

(E)

5.4 (D)

VRAAG 6

6.1 $P_{n+1} = P_n\left(1 + \dfrac{0,12}{12}\right) - m$

$P_{n+1} = 1,01P_n - m$ $P_o = x$

6.2 $P_1 = 1,01P_o - m$

$1\ 197\ 000 = 1,01P_o - m$

$P_3 = 1,01P_2 - m$

$P_3 = 1,01(1,01P_1 - m) - m$

$P_3 = 1,0201P_1 - 1,01m - m$

$1190\ 909,70 = 1,01^2(1197\ 000) - 2,01m$

$m = 15\ 000$

Totaal vir Module 3: 100 punte

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MODULE 4 MATRIKSE EN GRAFIEKTEORIE

VRAAG 1

1.1 (a) $\left(\begin{array}{ccc|c} 2 & 3 & -5 & 7 \\ 0 & 3 & -2 & 10 \\ 0 & -2 & 3 & 20 \end{array}\right)$

(b) $3R_3 + 2R_2 \quad \left(\begin{array}{ccc|c} 2 & 3 & -5 & 7 \\ 0 & 3 & -2 & 10 \\ 0 & 0 & 5 & 80 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 3 & -5 & 7 \\ 0 & 3 & -2 & 10 \\ 0 & 0 & 1 & 16 \end{array}\right) \quad Z = 16$

$R_2 + 2R_3 \quad \left(\begin{array}{ccc|c} 2 & 3 & -5 & 7 \\ 0 & 3 & 0 & 42 \\ 0 & 0 & 1 & 16 \end{array}\right) \quad y = 14 \qquad x = \frac{45}{2}$

1.2 (a) $\begin{pmatrix} 1 & 0 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x & -1 \\ -2 & 3 \end{pmatrix}$

$\begin{pmatrix} x+0 & -1+0 \\ 3x+4 & -3-6 \end{pmatrix} = \begin{pmatrix} x & -1 \\ 3x+4 & -9 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & -1 \\ 7 & -9 \end{pmatrix}\begin{pmatrix} -1 \\ 4 \end{pmatrix} = \begin{pmatrix} -5 \\ -43 \end{pmatrix}$

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VRAAG 2

2.1 (a) $\det = 3(x-6)-8(x-5)+2(6-5)$

$= 3x - 18 - 8x + 40 + 2$

$= -5x + 24$

(b) $\frac{1}{-5x+24}\begin{pmatrix} x-6 & -8x+12 & 6 \\ -x+5 & 3x-10 & -1 \\ 1 & 22 & -5 \end{pmatrix}$

$\frac{x-6}{-5x+24} = 1$

$22 = 2y$

$x = 5$ $y = 11$

2.2 $12 - 60i + 18i + 60i + 18i^2 + 12i$

$= -6 + 30i$

$\therefore x = -3 \;\; y = 10$

VRAAG 3

3.1 (a) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -3 & -1 \\ 1 & 1 & 2 \end{bmatrix}$

$a + b = -1$

$a + 3b = -3$

$c + 3d = 1$

$c + d = 1$

$a = 0, b = -1, c = 1$ en $d = 0$

$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 & 2 \\ 1 & 3 & 1 \end{bmatrix} = \begin{bmatrix} -1 & -3 & -1 \\ 1 & 1 & 2 \end{bmatrix}$

(b) Rotasie van 90 links om om die oorsprong.

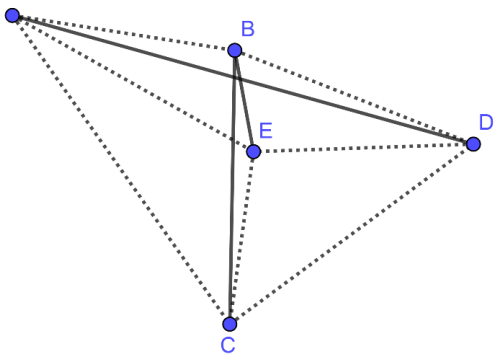
3.2 $\begin{pmatrix} \cos 30 & -\sin 30 \\ \sin 30 & \cos 30 \end{pmatrix}\begin{pmatrix} 4 & 12 & 0 \\ 8 & 16 & 8 \end{pmatrix} = \begin{pmatrix} -0.535 & 2.392 & -4 \\ 8.928 & 19.850 & 6.928 \end{pmatrix}$

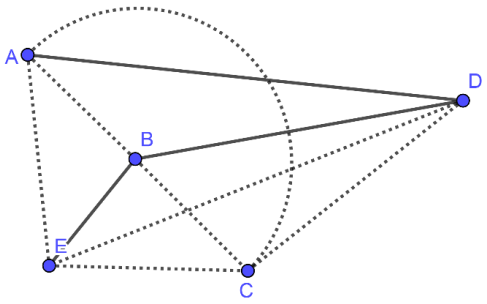
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VRAAG 4

4.1 (a) Grafiek 2 en 3

(b)





(c) Nee, hulle is nie.

4.2 (a) A en F het skakels van onewe graad

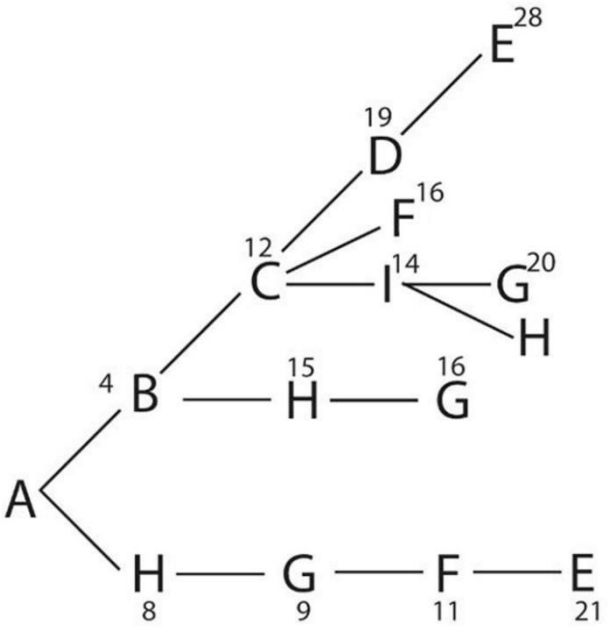
(b) ABE = 26, ADE = 25.5
$25.5 < 20 + 2x < 26$
$2.75 < x < 3$

(c) 11 skakels wat by A begin en by F eindig.
Tel dan terugkeer na A by.
Onewe nodus A en F.
Kortste pad om te herhaal is FE, ED en DA.
Totale gewig 208 + 4x.
A-B-C-A-D-C-E-B-F-D -E-F -E-D-A

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VRAAG 5

5.1



A H G F E = 21

5.2 AB – 4
AH – 8 / BC –8
HG – 1 / CI – 2
GF – 2 / CF – 4
FC – 4 / GF – 2
CI – 2 / HG – 1
CD – 7 / CD – 7
DE – 9 / DE – 9
= 37

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VRAAG 6

Laat n = k + 1

$$\begin{aligned}\begin{pmatrix}1 & -1\\ 0 & 2\end{pmatrix}^{k+1} &= \begin{pmatrix}1 & -1\\ 0 & 2\end{pmatrix}^{k}\begin{pmatrix}1 & -1\\ 0 & 2\end{pmatrix}\\ &= \begin{pmatrix}1 & 1-2^{k}\\ 0 & 2^{k}\end{pmatrix}\begin{pmatrix}1 & -1\\ 0 & 2\end{pmatrix}\\ &= \begin{pmatrix}1 & -1+2\left(1-2^{k}\right)\\ 0 & 2^{k+1}\end{pmatrix}\\ &= \begin{pmatrix}+1 & -1+2-2^{k+1}\\ 0 & 2^{k+1}\end{pmatrix}\\ &= \begin{pmatrix}+1 & 1-2^{k+1}\\ 0 & 2^{k+1}\end{pmatrix}\end{aligned}$$

Dus waar vir n = k + 1.

Deur die beginsel van volledige induksie waar vir alle $n \in \mathbb{N}$.

Totaal vir Module 4: 100 punte

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