INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
NOVEMBER 2024

FURTHER STUDIES MATHEMATICS (EXTENDED): PAPER II

MARKING GUIDELINES

Time: 1 hour 100 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

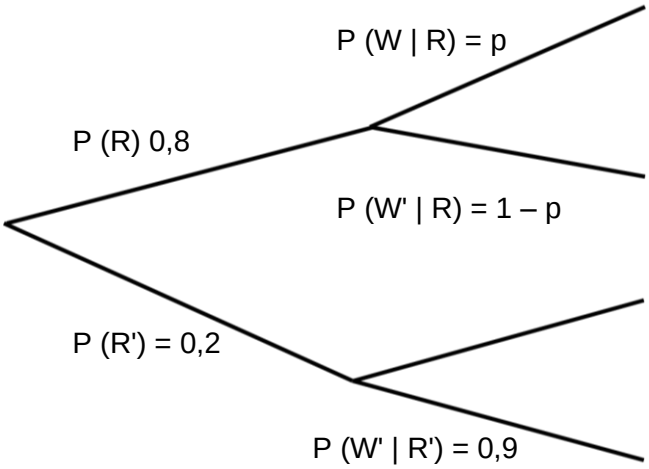
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MODULE 2 STATISTICS

QUESTION 1

1.1 (a)



(b) $0,8p+0,2(0,1) = 0,5$
$p = 0,6$

1.2 (a) A continuity correction is applied when a continuous distribution is used to approximate a discrete distribution.

(b) (1) $np = 2,8$
Since np < 5 hence no approximation should be used.

(2) $np = 28 > 5$; $nq = 42 > 5$ yes a normal approximation can be used,N (28;16,8)

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QUESTION 2

2.1 (a) $\bar{X} = \frac{8,3 + 9,9}{2} = 9,1$

$1,645 \times \frac{\sigma}{\sqrt{40}} = 9,9 - 9,1$

$\sigma = 3,08$

(b) $100(0,05) = 5$

2.2 (a) $H_0 : \mu = 300$

$H_1 : \mu < 300$

(b)

$\frac{298 - 300}{\frac{\sigma}{\sqrt{120}}} < -2,33$

$\sigma < 9,4$

QUESTION 3

(a) $X \sim N(6,5;1,15^2)$ ✓

$$P(X > 5) = P\left(Z > \frac{5-6,5}{1,15}\right) \checkmark$$
$$= P(Z > -1,3) \checkmark$$
$$= 0,5 + 0,4032 \checkmark$$
$$\approx 0,9 \checkmark$$

or calc:

$$P(X > 5) = P\left(Z > \frac{5-6,5}{1,15}\right)$$
$$= P(Z > -1,30435)$$
$$= 0,9039$$
$$\approx 0,9$$

(b) $P(X < t) = 0,05$ ✓

$$-1,645 = \frac{t-6,5}{1,15} \checkmark\checkmark$$
$$t = 4,6 \checkmark$$

$\therefore$ 4 years and 7 months. ✓

or calc

$$-1,6449 = \frac{t-6,5}{1,15}$$
$$= 4,6083$$

(c) $X \sim B(10;0,1)$ ✓✓

$$P(X \leq 1) = \binom{10}{0}(0,1)^0(0,9)^{10} + \binom{10}{1}(0,1)(0,9)^9 \checkmark\checkmark\checkmark$$
$$= 0,7361 \checkmark$$

Note: If they use p = 0,9032 (as per exact answer in (a)) then p(x ≤ 1) = 0,74847

(d) $X \sim B(9;0,1)$

$$= P(X = 1) \times 0,1$$
$$= \binom{9}{1}(0,1)(0,9)^8 \times 0,1 \checkmark\checkmark\checkmark$$
$$= 0,0387 \checkmark$$

(e) $$P(X < 3 \mid X < 5) = \frac{P(X < 3)}{P(X < 5)} \checkmark$$
$$= \frac{P(Z < \frac{3-6,5}{1,15})}{0,1 \checkmark}$$
$$= \frac{P(Z < -3,04)}{0,1} \checkmark$$
$$= \frac{0,5 - 0,49882}{0,1} \checkmark\checkmark$$
$$= 0,0118 \checkmark$$

With calc (ans = 0,01138)

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QUESTION 4

4.1 (a) $2p+q+0,3=1$

$\therefore 2p+q=0,7 \quad ...(1)$

$p+0,4+0,3+4q=2,45$

$p+4q=1,75 \quad ...(2)$

$\therefore p=0,15 \quad q=0,4$

(b) $P(X<2,45)=P(X=0)+P(X=1)+P(X=2)$

$=2(0,15)+0,2$

$=0,5$ ✓

4.2 (a) $A=\frac{\pi}{4}\left(\frac{2}{\sqrt{\pi}}\right)^2$

$=\frac{\pi}{4}\left(\frac{4}{\pi}\right)$

$=1$

$\therefore$ *a probability density function*

(b) $\cos A\hat{O}B=\frac{\frac{\sqrt{3}}{\pi}}{\frac{2}{\sqrt{\pi}}}$

$\cos A\hat{O}B=\frac{\sqrt{3}}{2}$

$A\hat{O}B=\frac{\pi}{6}$

(c) $P\left(X<\sqrt{\frac{3}{\pi}}\right)=A_{\text{sector YOB}}+A_{\Delta}$

$=\frac{1}{2}\left(\frac{2}{\sqrt{\pi}}\right)^2\left(\frac{\pi}{3}\right)+\frac{1}{2}\left(\sqrt{\frac{3}{\pi}}\right)\left(\frac{1}{\sqrt{\pi}}\right)$

$=0,9423$

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QUESTION 5

5.1

$$\frac{\binom{5}{1}\binom{3}{2}\binom{4}{2}+\binom{5}{1}\binom{3}{3}\binom{4}{1}+\binom{5}{1}\binom{3}{1}\binom{4}{3}}{\binom{12}{5}}$$

$$=\frac{85}{396}\,(0,21465)$$

(7)

5.2

$$\frac{44}{100}\times\frac{14}{99}+\frac{19}{100}\times\frac{25}{99}+\frac{25}{100}\times\frac{24}{99}$$
$$=0,1708$$

Or

$$\left(\frac{19}{100}\times\frac{39}{99}\right)+\left(\frac{25}{100}\times\frac{38}{99}\right)$$
$$=0,1708$$

Or

$$\left(\frac{3}{100}\times\frac{39}{99}\right)+\left(\frac{8}{100}\times\frac{39}{99}\right)+\left(\frac{25}{100}\times\frac{38}{99}\right)+\left(\frac{8}{100}\times\frac{39}{99}\right)$$
$$=0,1708$$

Or

$$\left(\frac{19}{100}\times\frac{39}{99}\right)+\left(\frac{25}{100}\times\frac{14}{99}\right)+\left(\frac{25}{100}\times\frac{24}{99}\right)$$
$$=0,1708$$

Total for Module 2: 100 marks

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MODULE 3 FINANCE AND MODELLING

QUESTION 1

Shailen takes out a loan of R15 000 now, and a further Rx rand 6 years from now. The loan is to be repaid by means of semi-annual instalments of Ry, starting in 6 months' time finishing in 10 years' time.

(a) Calculate x if y = 3 500 and the interest rate is constant at 10% per annum compounded semi-annually.

$$15\ 000\left(1+\frac{0{,}1}{2}\right)^{20}+x\left(1+\frac{0{,}1}{2}\right)^{8}=\frac{3\ 500\left[\left(1+\frac{0{,}1}{2}\right)^{20}-1\right]}{\frac{0{,}1}{2}}$$

$$x = R51\ 393{,}34$$

OR

$$1500+x\left(1+\tfrac{0{,}1}{2}\right)^{-12}=\frac{3500\left[1-\left(1+\tfrac{0{,}1}{2}\right)^{-20}\right]}{\frac{0{,}1}{2}}$$

(b) Calculate y, given that x = R10 000 and the interest is 10% per annum compounded semi-annually for the first 6 years and 12% per annum compounded semi-annually for the remaining 4 years.

$$15\,000\left(1+\frac{0{,}1}{2}\right)^{12}\left(1+\frac{0{,}12}{2}\right)^{8}+10\,000\left(1+\frac{0{,}12}{2}\right)^{8}$$

$$=\frac{y\left[\left(1+\frac{0{,}1}{2}\right)^{12}-1\right]\left(1+\frac{0{,}12}{2}\right)^{8}}{\frac{0{,}1}{2}}+\frac{y\left[\left(1+\frac{0{,}12}{2}\right)^{8}-1\right]}{\frac{0{,}12}{2}}$$

$$\therefore y = R\,1669{,}36$$

OR, bringing values to interest rate change:

$$15000\left(1+\frac{0{,}1}{2}\right)^{12}+10000=\frac{y\left[\left(1+\frac{0{,}1}{2}\right)^{12}-1\right]}{\frac{0{,}1}{2}}+\frac{y\left[1-\left(1+\frac{0{,}12}{2}\right)^{-8}\right]}{\frac{0{,}12}{2}}$$

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QUESTION 2

A loan is taken and is to be repaid by means of 120 monthly payments, starting after one month, with interest calculated at 12 % per annum, compounded monthly. The total interest paid over the course of the loan is R1 443 302,76.

(a) Calculate the monthly payment and show that the loan is R2 000 000.

(1) $$P = \frac{x\left[1-\left(1+\frac{0,12}{12}\right)^{-120}\right]}{\frac{0,12}{12}}$$

(2)

$$120x - P = 144\,3302,76$$

$$\therefore 120x - 144\,3302,76 = \frac{x\left[1-\left(1+\frac{0,12}{12}\right)^{-120}\right]}{\frac{0,12}{12}}$$

$$x = R28\,694,19$$

$$\therefore P = 120(28\,694,19) - 144\,3302,76$$

$$= R200\,0000$$

(b) Calculate the interest paid in the 81st month.

$$O.B_{80} = \frac{28\,694,19\left[1-\left(1+\frac{0,12}{12}\right)^{-40}\right]}{\frac{0,12}{12}}$$

$$= R942\,164,72$$

$$\therefore Interest = 942\,164,72 \times \frac{0,12}{12}$$

$$= R9\,421,65$$

OR

$$OB_{80} = 2000000\left(1+\tfrac{0,12}{12}\right)^{80} - \frac{28694,19\left[\left(1+\frac{0,12}{12}\right)^{80}-1\right]}{\frac{0,12}{12}}$$

$$= R9421,67$$

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QUESTION 3

A savings scheme, where interest is calculated monthly, is represented by the following recursive equation:

$F_{n+1} = aF_n + 1\,200\,(1{,}08)^n; \quad F_0 = b$

(a) After 1 month the balance is R3 230 and after 2 months it is R4 574,45. Calculate *a* and *b*.

The initial amount

$3\,230 = ab + 1\,200$

$2\,030 = ab$

$4\,574{,}45 = 3\,230a + 1\,200(1{,}08)$

$3\,278{,}45 = 3\,230a$

$a = 1{,}015$

$\therefore b = \dfrac{2\,030}{1{,}015}$

$= R\,2\,000$

(b) The effective rate, as a percentage correct to 2 decimal places.

i = 0,015

$r_{eff} = (1 + 0{,}015)^{12} - 1$

$= 19{,}56\%$

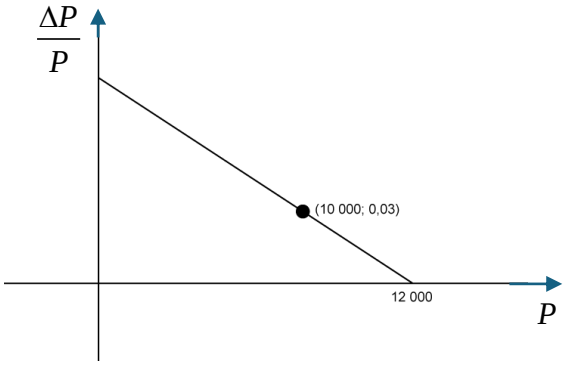
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QUESTION 4

A population, P_n, is growing logistically within an environment with a fixed carrying capacity, K. At the point at which the population is growing at its most rapid rate, P_n = 6 000.

Furthermore, when P = 10 000 then $\frac{\Delta P}{P} = 0,03$.

(a) Sketch the graph of $\frac{\Delta P}{P}$ against P, indicating at least two points on the graph.



(b) Prove that $P_{n+1} = 1,18P_n - 0,000015P_n^2$

$$m = \frac{0 - 0,03}{12\,000 - 10\,000}$$
$$= -0,000015$$
$$\therefore -\frac{0,03 - r}{10\,000} = -0,000015$$
$$\therefore r = 0,18$$

OR

$$\frac{\Delta P}{P} = -\frac{r}{1200}P + r$$
$$\therefore 0,03 = -\frac{r}{1200}(10000) + r$$
$$\therefore r = 0,18$$
$$P_{n+1} = P_n + 0,18P_n\left(1 - \frac{P_n}{12\,000}\right)$$
$$= P_n + 0,18P_n - \frac{0,18P_n^2}{12000}$$

(c) Determine $\frac{\Delta P}{P}$ when the growth rate is at its highest.

$$\frac{\Delta P}{P} = \frac{0,18}{2} \overset{\checkmark}{=} 0,09 \overset{\checkmark}{}$$

QUESTION 5

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There is a fascinating relationship between the great white shark and the Cape fur seal in the waters around Seal Island, off the coast of Cape Town. The sharks employ a hunting technique known as 'breaching', where they launch themselves vertically out of the water to catch seals swimming near the surface.





Great white sharks, of which 30% are thought to be reproducing, typically have a small litter every two years. The great whites have a long lifespan, but this can be taken to be 12 years in terms of their interaction with the island. There are currently 6 regular predators around the island. Each shark may eat up to 45 seal pups per year.



Female Cape fur seals typically produce a single pup every nine months, with a 15% pup mortality rate. Assume that one-quarter of the seal population is producing young and there are currently around 1 500 in the waters around the island, with a capacity for 1 650. With a prevalence of sharks, the equilibrium of seals is predicted to be 228. Consider a yearly cycle.

(a) Calculate the intrinsic growth rate of the seals, to 4 decimal places.

$$a = 0{,}85 \times 0{,}25 \times 1 \times \frac{12}{9}$$
$$= 0{,}2833$$

(b) Determine the rate of the successful attack of sharks on the seals.

$b.F_0.R_0 =$ decrease in seals

$$b \times 6 \times 1\,500 = 45 \times 6$$
$$b = 0{,}03$$

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(c) Calculate an estimate of the litter size of the great white shark

$$228 = \frac{1/12}{f \times 0{,}03}$$

$$\therefore f = 0{,}0122$$

$$fbR_0F_0 = 0{,}03 \times 0{,}0122 \times 6 \times 1500 = 0{,}3 \times 6 \times \frac{x}{2}$$

$$\therefore x = 3{,}66$$

Therefore 3 *or* 4.

(d) Determine the equilibrium number of sharks.

$$F_E = \frac{0{,}2833}{0{,}03}\left(1 - \frac{228}{1650}\right)$$

$$= 8{,}14$$

ie. about 8

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QUESTION 6

Two simultaneous recursive equations are given as follows:

$$y_{n+1} = \frac{3y_n}{5\,000}\left(1250 + x_n\right) \qquad y_0 = 100$$

$$x_{n+1} = \frac{x_n}{1000}\left(1\,500 - x_n - 20y_n\right) \qquad x_0 = 10$$

(a) Calculate $(x_2; y_2)$

$$y_1 = \frac{3\,(100)}{5000}\ (1250 + 100) = 75{,}6$$

$$x_1 = \frac{10}{1000}\ (1500 - 10 - 20\,(100)) = -5{,}1$$

(b) Determine the equilibrium values of x and y.

$$y_{n+1} = y_n \ ; x_{n+1} = x_n$$

$$5\,000 = 3\,(1250 + x_e)$$

$$x_e = 416\,\frac{2}{3}$$

$$1\,000 = 1500 - 416\tfrac{2}{3} - 20 \times y_e$$

$$y_e = 4\,\frac{1}{6}$$

Total for Module 3: 100 marks

MODULE 4 MATRICES AND GRAPH THEORY

QUESTION 1

1.1 (a) Rotation anti-clockwise by 90°

(b) A shear *y*-axis invariant

Scale factor of ½
OR

$x \Rightarrow x$

$y = y + \frac{1}{2}x$

(c) A stretch with the *x*-axis invariant by factor of 3.
then reflection in $y = -x$

OR

Rotate 90 degrees clockwise,
then a stretch with a scale factor of -3 with invariant y-axis.

1.2 (a) Find the transformation matrix that maps ΔABC to $\Delta A^1B^1C^1$

$$M \times \begin{bmatrix} 7 & 5 & 11 \\ 6 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 19 & 9 & 15 \\ 6 & 2 & 2 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$ or

$$\begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 7 & 5 & 11 \\ 6 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 19 & 9 & 15 \\ 6 & 2 & 2 \end{bmatrix}$$ pre-multiplying

$7 + 6k = 19$; $k = 2$

(b) Shear by factor 2 Invariant x-axis

(c) Ratio 1; Area remains unchanged.

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QUESTION 2

2.1 (a) square

(b) non-zero

2.2

$$\begin{bmatrix} 2 & 1 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ x & 1 \end{bmatrix} = \begin{bmatrix} 3x+2 & 7 \\ 7-x & 7 \end{bmatrix}$$

$4+x = 3x+2$

$x = 1$

2.3

$$\left[\begin{array}{ccc|c} 2 & -5 & 3 & 8 \\ 0 & 13 & -1 & -10 \\ 1 & 3 & 2 & -3 \end{array}\right] 2R_3 - R_1 \sim \left[\begin{array}{ccc|c} 2 & -5 & 3 & 8 \\ 0 & 13 & -1 & -10 \\ 0 & 11 & 1 & -14 \end{array}\right] R_3 + R_2$$

$$\sim \left[\begin{array}{ccc|c} 2 & -5 & 3 & 8 \\ 0 & 13 & -1 & -10 \\ 0 & 24 & 0 & -24 \end{array}\right]$$

$y = -1$
$z = -3$
$x = 6$

2.4 (a)

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 1 & 0 & 6 \end{vmatrix} = \begin{vmatrix} 4 & 5 \\ 0 & 6 \end{vmatrix} - 0 + \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix}$$

$= 24 - 0 - 2 = 22$

(b) Co-factor A $= \begin{bmatrix} 24 & 5 & -4 \\ -12 & 3 & 2 \\ -2 & -5 & 4 \end{bmatrix}$

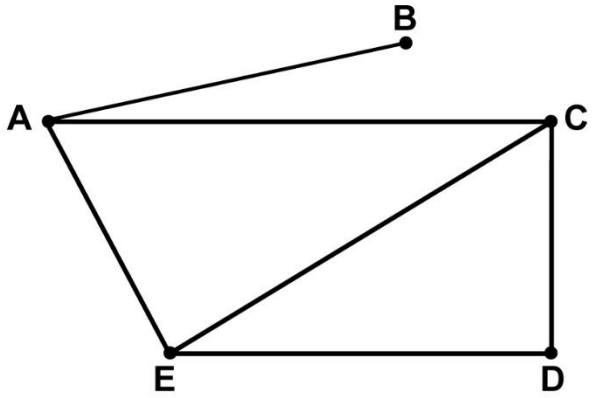
(c) $\frac{1}{22}\begin{bmatrix} 24 & -12 & -2 \\ 5 & 3 & -5 \\ -4 & 2 & 4 \end{bmatrix}$

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QUESTION 3

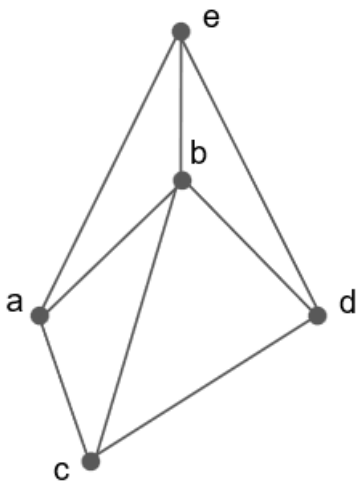
3.1 Pairs of edges that connect vertices: A; D; E; G

3.2



3.3 (a) Graph (i) has more edges that Graph (ii)

(b) (a, e) or (d, e)



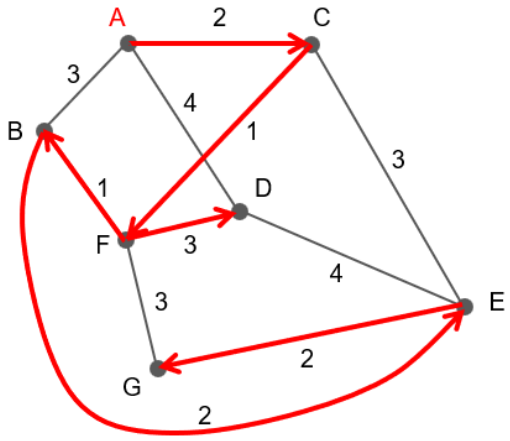
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QUESTION 4

4.1

A – C	2
C – F	1
F – B	1
B – E	2
E – G	2
F – D	3

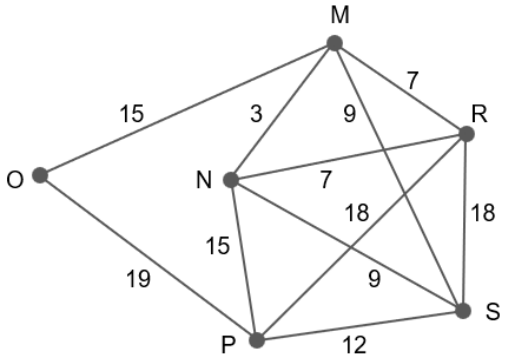
Total = 11



4.2 (a)

$$O \overset{15}{-} M \overset{3}{-} N \overset{7}{-} R \overset{18}{-} S \overset{12}{-} P \overset{19}{-} O$$

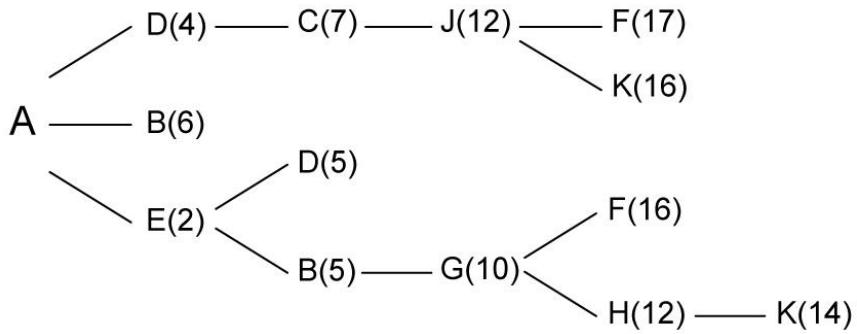
Total = 74



(b) For example: O – M – R – N – S – P – O = 69

QUESTION 5

(a)



A E B G H K (14)

(b) A E B G H K to J = 18
(14) (4)

(c) Triangle inequality $1 < BF < 11$

AEBFK is the shortest route
$\therefore BF + 8 < 14$ $BF < 6$

$1 < BF < 6$

Total for Module 4: 100 marks

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