NATIONAL SENIOR CERTIFICATE EXAMINATION
MAY 2022

MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours 150 marks

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NOTE:

- If a student answers a question more than once, only mark the FIRST attempt.
- Consistent Accuracy applies in all aspects of the marking memorandum.

SECTION A

QUESTION 1

(a)	$2x-7=4$ $x=\frac{11}{2}$	✓m squaring ✓ca $x=\frac{11}{2}$ (2)
(b)	$3x-1=\pm(x+1)$ $3x-1=x+1$ or $3x-1=-x-1$ $x=1$ or $x=0$ **OR** $9x^2-6x+1=x^2+2x+1$ $8x^2-8x=0$ $8x(x-1)=0$ $x=0$ or $x=1$	✓a ✓m $3x-1=\pm(x+1)$ ✓ca $x=1$ ✓ca $x=0$ (4) Multiplying out ✓a✓m ✓ca $x=1$ ✓ca $x=0$
(c)	$7^{2x}=7^{x-2}$ $\therefore 2x=x-2$ $\therefore x=-2$ **Alternate:** $7^{2x}-7^{x-2}=0$ $7^x.7^x-7^x.7^{-2}=0$ $7^x\left(7^x-\frac{1}{7^2}\right)=0$ $\therefore 7^x=0$ n/v or $7^x=7^{-2}$ $\therefore x=-2$	✓a $7^{2x}=7^{x-2}$ ✓ca $\therefore 2x=x-2$ ✓ca $\therefore x=-2$ ✓a $7^{2x}=7^{x-2}$ ✓ca $7^x\left(7^x-\frac{1}{7^2}\right)=0$ ✓ca $\therefore x=-2$ (3)
(d)	$\log_2(x^2-3x+10)=3$ $x^2-3x+10=2^3$ $x^2-3x+2=0$ $(x-2)(x-1)=0$ $x=2$ or $x=1$	✓a $x^2-3x+10=2^3$ ✓ca $(x-2)(x-1)=0$ ✓ca $x=2$ ✓ca $x=1$ (4)
(e)	$2x^2+3px+p^2=0$ $(2x+p)(x+p)=0$ $x=-\frac{1}{2}p$ or $x=-p$	✓m $(2x+p)(x+p)=0$ ✓a $x=-\frac{1}{2}p$ ✓a $x=-p$ (3) **[16]**

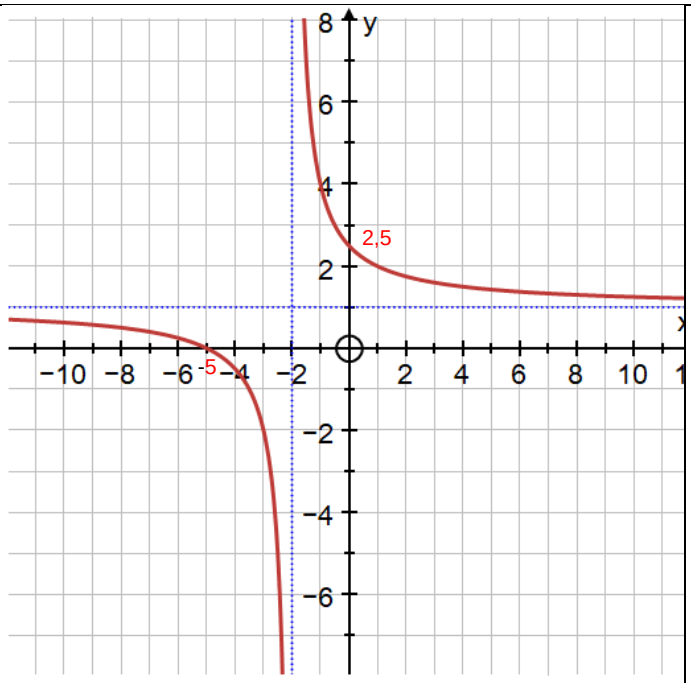
QUESTION 2

(a)(1)	$T_1 = 7$; $T_2 = 1$	✓a $T_1 = 7$ ✓ca $T_2 = 1$ (2)
(a)(2)	$S_\infty = \dfrac{7}{1-\frac{1}{7}}$ $S_\infty = \dfrac{49}{6}$	✓ca $S_\infty = \dfrac{7}{1-\frac{1}{7}}$ ✓ca $S_\infty = \dfrac{49}{6}$ (2)
(b)	2 5 6 5 2 Sequence 3 1 –1 –3 First difference –2 –2 –2 Constant second differ $2a = -2 \quad \therefore a = -1$ $3a + b = 3 \quad \therefore \quad b = 6$ $a + b + c = 2 \quad \therefore \quad c = -3$ $T_n = -n^2 + 6n - 3$	✓a First difference ✓a Constant second differ. ✓ca $a = -1$ ✓ca $b = 6$ ✓ca $c = -3$ (5) **[9]**

QUESTION 3

(a)	$P = x\left[\frac{1-(1+i)^{-n}}{i}\right]$ $1\,800\,000 = x\left[\frac{1-\left(1+\frac{9,25}{1200}\right)^{-30\times12}}{\frac{9,25}{1200}}\right]$ $x = R14\,808,16$	✓m Use of correct formula ✓a $\left(1+\frac{9,25}{1200}\right)$ ✓a $n = 360$ in Pv formula ✓ca $x = R14\,808,16$ (4)
(b)	OB = A – F $A = 1\,800\,000\left(1+\frac{9,25}{1200}\right)^{144}$ $A = 5\,438\,647,444$ $F = 14\,808,15766\left[\frac{\left(1+\frac{9,25}{1200}\right)^{144}-1}{\frac{9,25}{1200}}\right]$ $F = 3\,883\,363,245$ $\text{OB} = 1\,555\,284,20$ **OR** OB $= 14\,808,16\frac{1-\left(1+\frac{9,25}{1\,200}\right)^{-216}}{\frac{9,25}{1\,200}}$ $= \text{R}1\,555\,284,45$	✓a interest rate ✓a for n ✓a Correct formula ✓ca Substitution ✓ca answer (5) ✓a interest rate ✓a for n ✓a Correct formula ✓ca Substitution ✓ca answer **[9]**

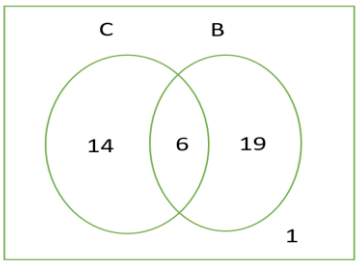
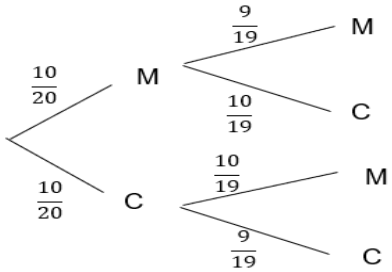
QUESTION 4

(a)(1)		✓ Shape ✓ Vertical asymptote ✓ Horizontal asymptote ✓ *x*-intercept ✓ *y*-intercept (5)
(a)(2)	$\mathbb{R}-\{-2\}$ **OR** $(-\infty;-2)\cup(-2;\infty)$ **OR** $\{x: x<-2 \text{ or } x>-2\}$	(1)
(b)(1)	$y=3^x$ For inverse: let $x=3^y$ $g(x)=\log_3 x$	✓m $x=3^y$ ✓a $g(x)=\log_3 x$ (2)
(b)(2)(i)	$x \in R$	✓a $x \in R$ (1)
(b)(2)(ii)	$\therefore 0<x\leq 1$	✓✓a (2) **[11]**

QUESTION 5

(a)	$f(x)=-2x^2+3x$ $f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{-2(x+h)^2+3(x+h)-(-2x^2+3x)}{h}$ $f'(x)=\lim_{h\to 0}\frac{-2x^2-4xh-2h^2+3x+3h+2x^2-3x}{h}$ $f'(x)=\lim_{h\to 0}\frac{-4xh-2h^2+3h}{h}$ $f'(x)=\lim_{h\to 0}\frac{h(-4x-2h+3)}{h}$ $f'(x)=\lim_{h\to 0}(-4x-2h+3)$ $=-4x+3$	✓ $\frac{-2(x+h)^2+3(x+h)-(-2x^2+3x)}{h}$ ✓ Squaring and distributing ✓ Factorisation ✓ Notation ✓ Sub. to get: $=-4x+3$ (5)
(b)(1)	$y=2x^3-4x+5$ $\frac{dy}{dx}=6x^2-4$	✓a $6x^2$ ✓ca -4 (2)
(b)(2)	$y=x^{-\frac{2}{5}}$ $\frac{dy}{dx}=-\frac{2}{5}x^{-\frac{7}{5}}$	$y=x^{-\frac{2}{5}}$ $\frac{dy}{dx}=-\frac{2}{5}x^{-\frac{7}{5}}$ (2)
(c)	For $f'(x)>0$ $f'(x)=-x^2+2x+3=0$ $x=3$ or $x=-1$ Increasing for: $-1\le x\le 3$ Also accept <	✓a $f'(x)=-x^2+2x+3$ ✓a $f'(x)=0$ ✓ca $x=3$ or $x=-1$ ✓ca✓ca $-1\le x\le 3$ (5) **[14]**

QUESTION 6

(a)(1)	No, There are 6 students playing both cricket and basketball.	✓a No ✓a Explanation (2)
(a)(2)	$P(C \text{ or } B) = P(C) + P(B) - P(C \text{ and } B)$ $P(C \text{ or } B) = \frac{20}{40} + \frac{25}{40} - \frac{6}{40}$ $= \frac{39}{40}$ Alternate:  $P(C \text{ or } B) = \frac{39}{40}$	✓a $\frac{20}{40} + \frac{25}{40}$ ✓a $-\frac{6}{40}$ ✓a $= \frac{39}{40}$ (3)
(b)(1)	 	✓a correct diagram ✓a correct M probabilities ✓a correct C probabilities (3)
(b)(2)	P(M and C) OR P(C and M) $= \frac{10}{20} \times \frac{10}{19} + \frac{10}{20} \times \frac{10}{19}$ $= \frac{10}{19}$	✓a $\frac{10}{20} \times \frac{10}{19}$ ✓a $\frac{10}{20} \times \frac{10}{19}$ ✓ca $\frac{10}{19}$ (3)
(c)	$\frac{4!}{6!}$ $= \frac{4!}{6 \times 5 \times 4!}$ $= \frac{1}{30}$	✓a $\frac{4!}{6!}$ ✓a $= \frac{4!}{6 \times 5 \times 4!}$ ✓a $= \frac{1}{30}$ (3) **[14]**

73 marks

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SECTION B

QUESTION 7

(a)	$3\,500\,000 = P\left(1+\frac{4,5}{100}\right)^{13}$ $P = \text{R}1\ 974\ 950,74$	✓ Correct formula ✓a $3\,500\,000 = P\left(1+\frac{4,5}{100}\right)^{13}$ ✓ca $P = \text{R}1\ 974\ 950,74$ (3)
(b)	$1\,800\,000\left(1+\frac{5}{100}\right)^{n} = 1974\,950,74\left(1+\frac{4,5}{100}\right)^{n}$ $\frac{\left(1+\frac{5}{100}\right)^{n}}{\left(1+\frac{4,5}{100}\right)^{n}} = \frac{1974\,950,74}{1800\,000}$ $\left(\frac{1,05}{1,045}\right)^{n} = 1,0971948...$ $n = 19,4$ Year: 2027	✓ca✓m $1\,800\,000\left(1+\frac{5}{100}\right)^{n} = 1974\,950,74\left(1+\frac{4,5}{100}\right)^{n}$ ✓ca $\frac{\left(1+\frac{5}{100}\right)^{n}}{\left(1+\frac{4,5}{100}\right)^{n}} = \frac{1974\,950,74}{1800\,000}$ ✓ca $n = 19,4$ ✓ca2027 (5) **[8]**

QUESTION 8

(a)	$7 + x + y = -2x - 7$ $y = -3x - 14$ $\frac{x}{7} = \frac{-3x-14}{x}$ $x^2 + 21x + 98 = 0$ $x = -7$ or $x = -14$ $y = 7$ or $y = 28$	✓a $7 + x + y = -2x - 7$ ✓a ✓a $\frac{x}{7} = \frac{-3x-14}{x}$ ✓ca✓ca $x = -7$ or $x = -14$ ✓ca✓ca $y = 7$ or $y = 28$ (7)
(b)	$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ $160 = \frac{n}{2}\left[2a + (n-1)(5)\right]$ $320 = 2an + 5n^2 - 5n$ $725 = \frac{2n}{2}\left[2a + (2n-1)(5)\right]$ $725 = 2an + 10n^2 - 5n$ $405 = 5n^2$ $n = 9$	✓a $160 = \frac{n}{2}\left[2a + (n-1)(5)\right]$ ✓ca $320 = 2an + 5n^2 - 5n$ ✓a $725 = \frac{2n}{2}\left[2a + (2n-1)(5)\right]$ ✓ca $725 = \frac{2n}{2}\left[2a + (2n-1)(5)\right]$ ✓ca $405 = 5n^2$ ✓ca $n = 9$ (6) **[13]**

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QUESTION 9

(a)	$y = a(x+1)^2 + q$ sub. $(-4;0)$ $0 = a(-4+1)^2 + q$ $0 = 9a + q$ .. eq 1 $y = a(x+1)^2 + q$ sub. $(-2;4)$ $4 = a(-2+1)^2 + q$ $4 = a + q$ ... eq.2 $a = -\frac{1}{2}$ $q = \frac{9}{2}$ $y = -\frac{1}{2}(x+1)^2 + \frac{9}{2}$ $y = -\frac{1}{2}x^2 - x + 4$ $\therefore a = -\frac{1}{2}$; $b = -1$; $c = 4$ **OR** x- intercepts are –4 and 2 $y = a(x+4)(x-2)$ Substitute (–2;4) $4 = a(-2+4)(-2-2)$ $\therefore a = -\frac{1}{2}$ $y = -\frac{1}{2}(x+4)(x-2)$ $y = -\frac{1}{2}x^2 - x + 4$ $\therefore a = -\frac{1}{2}$; $b = -1$; $c = 4$	$y = a(x+1)^2 + q$ ✓ca $0 = 9a + q$ .. eq 1 ✓ca $4 = a + q$ ... eq.2 ✓ca $a = -\frac{1}{2}$ ✓ca $b = -1$ ✓ca $c = 4$ (5) ✓a $y = a(x+4)(x-2)$ Correct substitution ✓ca ✓ca $a = -\frac{1}{2}$ ✓ca $b = -1$ ✓ca $c = 4$
(b)(1)	$f(x) = -\frac{1}{2}(x^2 + 2x - 8)$ $f(x) = -\frac{1}{2}\left[(x+1)^2 - 8 - 1\right]$ $f(x) = -\frac{1}{2}\left[(x+1)^2 - 9\right]$ $f(x) = -\frac{1}{2}(x+1)^2 + \frac{9}{2}$	✓a $f(x) = -\frac{1}{2}(x^2 + 2x - 8)$ ✓a $f(x) = -\frac{1}{2}\left[(x+1)^2 - 8 - 1\right]$ ✓a $f(x) = -\frac{1}{2}\left[(x+1)^2 - 9\right]$ (3)
(b)(2)	$-2 < x < 0$ and $x \neq -1$	✓a $-2 < x < 0$ ✓a ✓a $x \neq -1$ (3)
(c)	X-int of g: $x = -8$ X-int of f: $x = 2$ or $x = -4$ $x \in [-8;-4] \cup [2;\infty)$	✓a ✓a $x \in [-8;-4]$ ✓ca $[2;\infty)$ (3)

(d)	Top – Bottom $= x+8-\left[-\frac{1}{2}(x+1)^2+\frac{9}{2}\right]$ $= x+8+\frac{1}{2}x^2+x-4$ $=\frac{1}{2}x^2+2x+4$ $\frac{dy}{dx}=x+2$ $\therefore x=-2$ Top – Bottom $=\frac{1}{2}x^2+2x+4$ ... sub. $x=-2$ $\therefore$ Max. 2 units	✓a $= x+8-\left[-\frac{1}{2}(x+1)^2+\frac{9}{2}\right]$ ✓ca $=\frac{1}{2}x^2+2x+4$ ✓ca $\frac{dy}{dx}=x+2$ ✓ca $\therefore x=-2$ ✓ca $\frac{1}{2}x^2+2x+4$ ...sub. $x=-2$ ✓ca $\therefore$ Max. 2 units (5) **[19]**

QUESTION 10

$y=a(-x)^2-7(-x)+c$ $\therefore b=7$ $-\frac{b}{2a}=1$ $\therefore a=-\frac{7}{2}$ $f(x)=-\frac{7}{2}x^2+7x+c$ sub $(4;-20)$ $-20=-\frac{7}{2}(4)^2+7(4)+c$ $c=8$ $\therefore f(x)=-\frac{7}{2}x^2+7x+8$	✓a $y=a(-x)^2-7(-x)+c$ ✓a $\therefore b=7$ ✓ca $-\frac{b}{2a}=1$ ✓ca $\therefore a=-\frac{7}{2}$ ✓ca $-20=-\frac{7}{2}(4)^2+7(4)+c$ ✓ca $c=8$
Alternative: $f(x)=a(-x)^2-7(-x)+c$ $f(x)=ax^2+7x+c$ $f'(x)=2ax+7$ for TP sub. $(x=1)$ $2a(1)+7=0$ $\therefore a=-\frac{7}{2}$ $\therefore b=7$ $f(x)=-\frac{7}{2}x^2+7x+c$ sub $(4;-20)$ $-20=-\frac{7}{2}(4)^2+7(4)+c$ $c=8$ $\therefore f(x)=-\frac{7}{2}x^2+7x+8$	✓a $y=a(-x)^2-7(-x)+c$ ✓ca $f'(x)=2ax+7$ ✓ca $\therefore a=-\frac{7}{2}$ ✓a $\therefore b=7$ ✓ca $-20=-\frac{7}{2}(4)^2+7(4)+c$ ✓ca $c=8$ **[6]**

QUESTION 11

(a)	$-3^3+5\times3^2-3\times5+d=0$ $d=9$ **Alternate:** $y=-1(x-3)(x-3)(x+p)$ $y=-1(x^2-6x+9)(x+p)$ $y=-1(x^3-6x^2+9x+px^2-6px+9p)$ $y=-x^3+(6-p)x^2-(9-6p)x-9p$ $\therefore 9-6p=3$ $p=1$ $\therefore d=-9(p)=-9$	✓a $-3^3+5\times3^2-3\times5+d=0$ ✓ca $d=9$ $-1(x-3)(x-3)(x+p)$ $-x^3+(6-p)x^2-(9-6p)x-9p$ ✓a $\therefore 9-6p=3$ $p=1$ ✓ca $\therefore d=-9(p)=-9$ (2)
(b)	$f'(x)=-3x^2+10x-3$ $f''(x)=-6x+10=0$ $-6x=-10$ $x=\frac{5}{3}$ Gradient = $-3\left(\frac{5}{3}\right)^2+10\times\frac{5}{3}-3$ $=\frac{16}{3}$	✓a $f'(x)=-3x^2+10x-3$ ✓ca $f''(x)=-6x+10=0$ ✓ca $x=\frac{5}{3}$ ✓ca $-3\left(\frac{5}{3}\right)^2+10\times\frac{5}{3}-3$ ✓ca $=\frac{16}{3}$ (5)
(c)	$x<\frac{5}{3}$ Allow $x\le\frac{5}{3}$	✓ca $x<\frac{5}{3}$ (1)
(d)	$f(x)-k+3=0$ $f(x)=k-3$ $f'(x)=-3x^2+10x-3=0$ $x=3$ or $x=\frac{1}{3}$ When $x=\frac{1}{3}$; $y=-\frac{256}{27}$ $\therefore -\frac{256}{27}\le k-3\le0$ $\therefore -\frac{175}{27}\le k\le3$	✓m derivative =0 ✓ca $x=3$ or $x=\frac{1}{3}$ substitution of one third ✓ca $y=-\frac{256}{27}$ ✓ca $\therefore -\frac{256}{27}\le k-3\le0$ ✓ca $\therefore -\frac{175}{27}\le k\le3$ (5) **[13]**

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QUESTION 12

(a)	$f'(0)=15$	✓a $f'(0)=15$ (1)
(b)	$f'(a)=0$ $\therefore a=10$	✓a $\therefore a=10$ (1)
(c)	$m=\frac{15-0}{0-10}=-\frac{3}{2}$ $f'(x)=-\frac{3}{2}x+15$ $\therefore f'(1)=13{,}5$	✓m ✓a ✓ca for equation ✓ca for gradient (4)
(d)	Graph is concave down – no p.o.i	✓a (1) **[7]**

QUESTION 13

(a)	Let: $x^2+2tx-t=x-\frac{5}{2}$ $x^2+(2t-1)x+\left(\frac{5}{2}-t\right)=0$ $\Delta=(2t-1)^2-4(1)\left(\frac{5}{2}-t\right)$ $4t^2-9<0$ Crit values: $-\frac{3}{2};\frac{3}{2}$ Will never touch for: $-\frac{3}{2}<t<\frac{3}{2}$	✓a $x^2+2tx-t=x-\frac{5}{2}$ ✓a $\Delta=(2t-1)^2-4(1)\left(\frac{5}{2}-t\right)$ ✓a $\Delta<0$ or any method ✓a $4t^2-9<0$ ✓ca Crit values: $-\frac{3}{2};\frac{3}{2}$ ✓ca $-\frac{3}{2}<t<\frac{3}{2}$ (6)
(b)	$\frac{500}{x}-\frac{500}{x+3}=40$ $40x^2+120x-1500=0$ $2x^2+6x-75=0$ $x=4{,}8$ or $x=-7{,}8$ Original time: $\frac{500}{4{,}805}$ $=104{,}1$ seconds	✓a $\frac{500}{x}-\frac{500}{x+3}=40$ ✓a $40x^2+120x-1500=0$ ✓a ✓a $x=4{,}8$ or $x=-7{,}8$ ✓a $\frac{500}{4{,}805}$ $=104{,}1$ seconds (5) **[11]**

77 marks

Total: 150 marks

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