INTERNASIONALE SEKONDÊRE SERTIFIKAAT-EKSAMEN
NOVEMBER 2022

VERDERE STUDIES WISKUNDE (STANDAARD): VRAESTEL I

NASIENRIGLYNE

Tyd: 2 uur 200 punte

Hierdie nasienriglyne word voorberei vir gebruik deur eksaminatore en hulpeksaminatore. Daar word van alle nasieners vereis om 'n standaardiserings-vergadering by te woon om te verseker dat die nasienriglyne konsekwent vertolk en toegepas word tydens die nasien van kandidate se skrifte.

Die IEB sal geen gesprek aanknoop of korrespondensie voer oor enige nasienriglyne nie. Daar word toegegee dat verskillende menings rondom sake van beklemtoning of detail in sodanige riglyne mag voorkom. Dit is ook voor die hand liggend dat, sonder die voordeel van bywoning van 'n standaardiseringsvergadering, daar verskillende vertolkings mag wees oor die toepassing van die nasienriglyne.

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VRAAG 1

1.1 (a) $\ln(x-2)=1$ of $\ln(x-2)=-1$

$\therefore x-2=e$ of $x-2=e^{-1}$

$\therefore x=e+2$ of $x=e^{-1}+2$

$\therefore x=4,72$ of $x=2,37$

(b) $\therefore e^x(e-1)=12$

$\therefore e^x=\dfrac{12}{e-1}$

$\therefore x=\ln\dfrac{12}{e-1}=1.94$

(c) $\therefore -2|2x+4|\geq -16$

$\therefore |2x+4|\leq 8$

$\therefore -8\leq 2x+4\leq 8$

$\therefore -12\leq 2x\leq 4$

$\therefore -6\leq x\leq 2$

1.2 $\dfrac{15-5ai}{a+2i}\times\dfrac{a-2i}{a-2i}=\dfrac{15a-30i-5a^2i+10ai^2}{a^2-4i^2}=-1-7i$

$\therefore \dfrac{5a}{a^2+4}+\dfrac{-30-5a^2}{a^2+4}i=-1-7i$

$\therefore \dfrac{5a}{a^2+4}=-1$

$\therefore 5a=-a^2-4$

$\therefore a^2+5a+4=0$

$\therefore (a+1)(a+4)=0$

$\therefore a=-1$ of $a=-4$

maar kontrolering toon dat $a=-1$ die enigste opsie is wat die korrekte imaginêre deel genereer

$\therefore a=-1$

ALTERNATIEWE OPLOSSING 1

$\therefore 15-5ai=(-1-7i)(a+2i)$

$\therefore 15-5ai=-a-7ai-2i-14i^2$

$\therefore 15-5ai=13-(7a+2)i$

$\therefore 5a=7a+2$

$\therefore -2a=2$

$\therefore a=-1$

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ALTERNATIEWE OPLOSSING 2

$\therefore 15-5ai=(-1-7i)(a+2i)$
$15=-a-14i^2$
$\therefore a=-1$

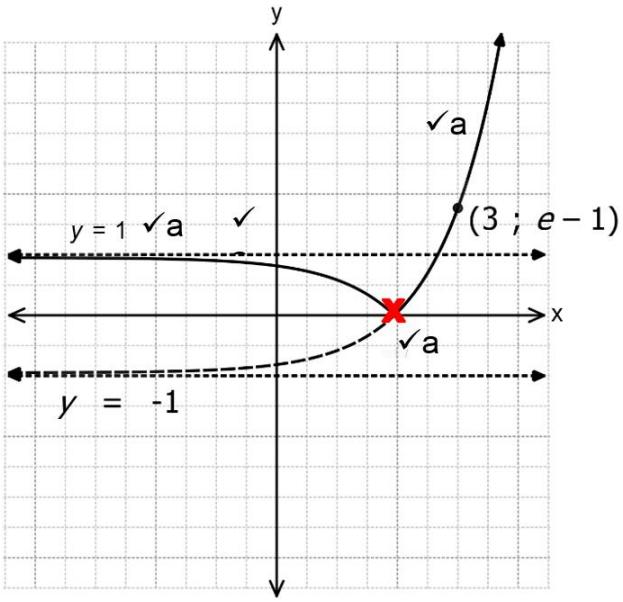
1.3 (a) indien $2+i$ 'n wortel is, dan is $2-i$ 'n wortel
som van wortels is 4 en produk van wortels is 5
$\therefore f(x)=\left(x^2-4x+5\right)$ is 'n faktor
$\therefore f(x)=\left(x^2-4x+5\right)\left(x^2+x-20\right)$
$\therefore f(x)=\left(x^2-4x+5\right)(x+5)(x-4)$

(b) $\left(x^2-4x+5\right)(x+5)(x-4)=0$
$\therefore x=2+i$ of $2-i$ of -5 of 4

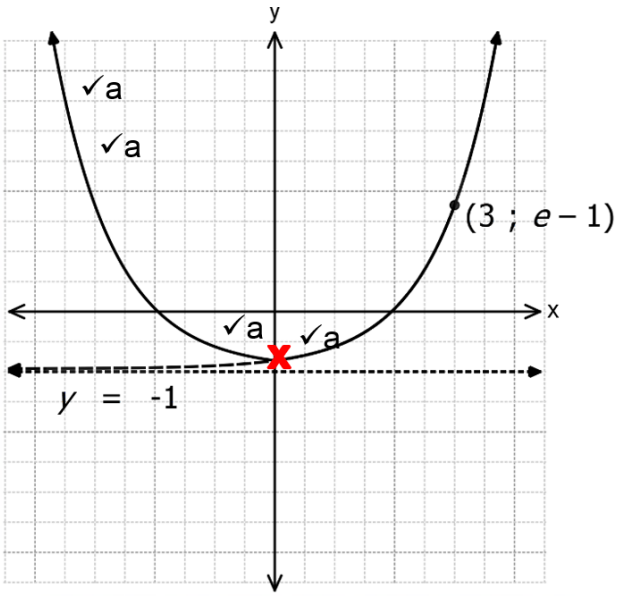
VRAAG 2

2.1 (a) $c=-1$
$e-1=e^{3+p}-1$
$\therefore e=e^{3+p}$
$\therefore p=-2$

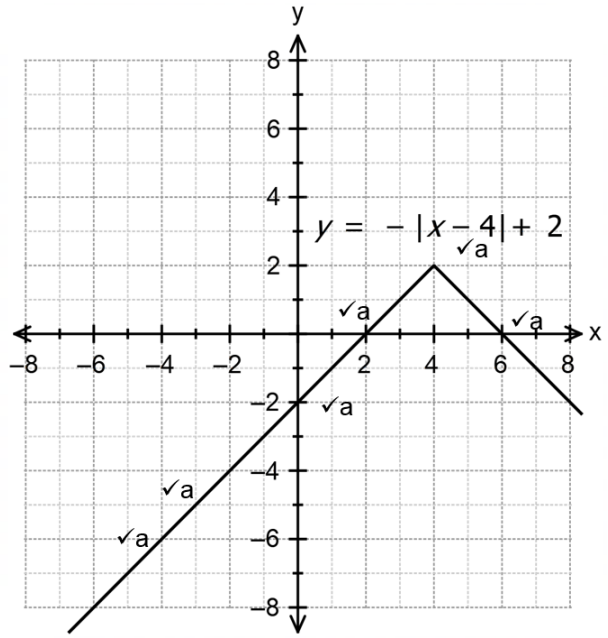
(b) (i)



(ii)



2.2



VRAAG 3

3.1 $ax - 8 = 0$ wanneer $x = 2$

$\therefore a = 4$

nou: $2x^2 + bx - 10 = (4x - 8)\left(\frac{1}{2}x + 4\right) + R$

$\therefore 2x^2 + bx - 10 = 2x^2 + 12x - 32 + R$

$\therefore b = 12$

3.2
- 'n y-afsnit van 3
- x-afsnitte van –2 en 3
- Twee vertikale asimptote waarvan een $x=1$ moet wees
- 'n Horisontale asimptoot van $y=-2$

$$y=\frac{-2(x+2)(x-3)}{(x-1)(x+4)}$$

3.3 $$g'(x)=\frac{(2x+3)(3x-13)-3(x^2+3x-10)}{(3x-13)^2}=0$$

$$\therefore 3x^2-26x-9=0$$

$$\therefore (3x+1)(x-9)=0$$

$$\therefore x=-\frac{1}{3} \text{ of } 9$$

$$\therefore \left(-\frac{1}{3};0{,}78\right) \text{ en } (9;7)$$

VRAAG 4

4.1 (a) $p=2$ (2)

(b) f moet kontinu wees by $x=-2$

nou: $\lim\limits_{x\to -2^+} f(x)=1$

$\therefore \lim\limits_{x\to -2^-} f(x)=1$

$\therefore -2m+c=1$

ook: $\lim\limits_{x\to -2^+} f'(x)=\lim\limits_{x\to -2^+}(-2x)=4$

$\therefore \lim\limits_{x\to -2^-} f'(x)=4$

$\therefore m=4$

$\therefore c=9$

4.2 (a) $$x^2+10y^2+14x+16y=2$$

$$\therefore 2x+20y\frac{dy}{dx}+14+16\frac{dy}{dx}=0$$

$$\therefore \frac{dy}{dx}(20y+16)=-2x-14$$

$$\therefore \frac{dy}{dx}=\frac{-2x-14}{20y+16}$$

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(b) $\frac{dy}{dx} = \frac{-2x-14}{20y+16}$

$\left.\frac{dy}{dx}\right|_{(-2;1)} = \frac{-10}{36} = -\frac{5}{18}$

$\therefore$ gradiënt van loodlyn $= \frac{18}{5}$

$\therefore y - 1 = \frac{18}{5}(x+2)$

$\therefore y = \frac{18}{5}x + \frac{41}{5}$ (hierdie lyn hoef nie getoon te word nie)

VRAAG 5

5.1 (a) $f(x) = x^3 - 2x + 2 = 0$

$\therefore f'(x) = 3x^2 - 2$

$\therefore x_{n+1} = x_n - \frac{x_n{}^3 - 2x_n + 2}{3x_n{}^2 - 2}$ ✓a

(b) Die raaklyn by $x - 1$ sny die x-as by 0 en die raaklyn by $x = 0$ sny die x-as by 1, dus is die antwoorde siklies in plaas van konvergerend.

Let daarop dat ons enige kommentaar wat "siklies"/lus voorstel, sal aanvaar.

(c) $x = -1{,}76929$

5.2 (a) $\cos\theta = \frac{3^2 + 4^2 - 6^2}{2(3)(4)} = -\frac{11}{24}$

$\therefore \theta = 2{,}047$

(b) gearseerde opp = opp van sektor − opp van driehoek

$= \frac{5^2(2{,}047)}{2} - \frac{1}{2}(3)(4)\sin 2{,}047$

$= 20{,}25\ \text{m}^2$

20,26 indien 'n mens die afgeronde 2,047 gebruik

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VRAAG 6

$\sum_{i=1}^{n}(3i-1)(3i+2) = 3n^3 + 6n^2 + n$

indien $n = 1$, is LK $= 10$ en RK $= 10$

dus is dit waar vir $n = 1 -$ beskou $n = k + 1$

Neem aan waar vir $n = k$, naamlik

$(2)(5) + (5)(8) + (8)(11) + ... + (3k-1)(3k+2) = k^3 + 6k^2 + k \quad (*)$

Indien nou $n = k + 1$, dan:

$$(2)(5) + (5)(8) + (8)(11) + ... + (3k-1)(3k+2) + (3k+2)(3k+5) = 3k^3 + 6k^2 + k + (3k+2)(3k+5)$$
$$= 3k^3 + 6k^2 + k + 9k^2 + 21k + 10$$
$$= 3k^3 + 9k^2 + 9k + 3 + 6k + 12k + 6 + k + 1$$

maar dit is net $(*)$ met $k + 1$ vir k

dus is dit waar vir $n = k + 1$

$\therefore$ volgens die beginsel van volledige induksie is dit waar vir $n \in \mathbb{N}$

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VRAAG 7

7.1 $10\,200 = 10\,000e^{k}$

$$\therefore e^{k} = \frac{10\,200}{10\,000}$$

$$\therefore k = \ln\frac{102}{100} = 0.0198$$

7.2 $y = y_0 e^{kt}$

$$\therefore \frac{y}{y_0} = e^{kt}$$

$$\therefore kt = \ln\frac{y}{y_0}$$

$$\therefore t = \frac{\ln\frac{y}{y_0}}{k}$$

$$t = \frac{\ln 10}{0{,}0198} = 116{,}29$$

$$\therefore t = 117 \text{ maande}$$

VRAAG 8

$$\text{gearseerde oppervlakte} = \int_1^4 f(x)dx - \int_1^4 g(x)dx$$

$$\therefore \frac{21}{2} = \frac{14}{3} - \int_1^4 f(x) - kx - k\ dx$$

$$\therefore \frac{21}{2} = \frac{14}{3} - \left(\int_1^4 f(x)dx - \int_1^4 kx + k\ dx\right)$$

$$\therefore \frac{21}{2} = \frac{14}{3} - \frac{14}{3} + \left[\frac{kx^2}{2} + kx\right]_1^4$$

$$\therefore \frac{21}{2} = 12k - \left(\frac{k}{2} + 1\right)$$

$$\therefore 21 = 24k - k - 2$$

$$23 = 23k$$

$$\therefore k = 1$$

ALTERNATIEWE METODE

$$\frac{21}{2} = \int_1^4 f(x) - g(x)\ dx$$
$$\frac{21}{2} = \int_1^4 f(x) - f(x) + kx + k\ dx$$
$$\frac{21}{2} = \int_1^4 kx + k\ dx$$
$$\frac{21}{2} = \left[\frac{kx^2}{2} + kx\right]_1^4$$
$$\frac{21}{2} = 8k + 4k - \frac{k}{2} - k$$
$$21 = 16k + 8k - k - 2k$$
$$k = 1$$

VRAAG 9

9.1 $\int -\text{cosec}^2\theta \cot\theta\, d\theta$

$= \int \text{cosec}\theta\left(-\text{cosec}\theta\cot\theta\right)d\theta$

$= \dfrac{\text{cosec}^2\theta}{2} + c$

9.2 $\int \dfrac{3x}{\sqrt{2x+5}\ \checkmark\text{a}}dx$

$= 3\int x\left(2x^2+5\right)^{-\frac{1}{2}}dx$

$= \dfrac{3\left(2x^2+5\right)^{\frac{1}{2}}}{\frac{1}{2}\times 4} + c$

$= \dfrac{3\left(2x^2+5\right)^{\frac{1}{2}}}{2} + c$

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ALTERNATIEF

$= 3\int x\left(2x^2+5\right)^{-\frac{1}{2}}dx$

laat $u = 2x^2+5$ dan $\frac{du}{dx} = 4x$ dus $dx = \frac{du}{4xdx}$

$= \frac{3}{4}\int u^{-\frac{1}{2}}du$

$= \frac{3}{4}\left(\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right)+c$

$= \frac{3u^{\frac{1}{2}}}{2}+c$

$= \frac{3\left(2x^2+5\right)^{\frac{1}{2}}}{2}+c$

9.3 $\frac{2x^2+3x+8}{x^2-x-6}$

$= \frac{2\left(x^2-x-6\right)+5x+20}{x^2-x-6}$

$= \frac{2\left(x^2-x-6\right)}{x^2-x-6}+\frac{5x+20}{x^2-a-6}$

$= 2+\frac{5x+20}{(x-3)(x+2)}$

nou $\frac{5x+20}{(x-3)(x+2)} = \frac{A}{x-3}+\frac{B}{x+2}$

$A = 7$ en $B = -2$ beide deur oordekkingsmetode

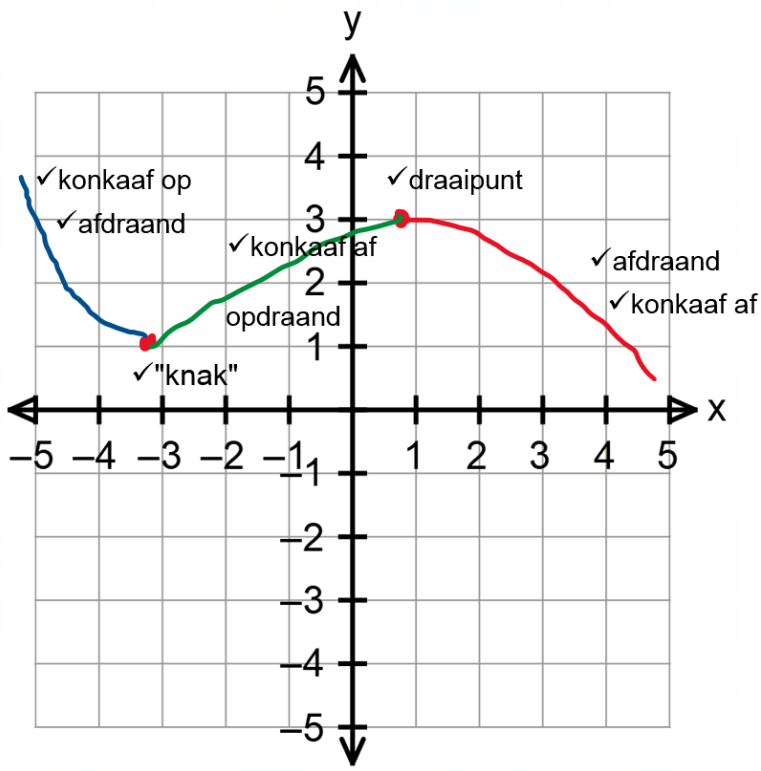
$\therefore \int \frac{2x^2+3x+8}{x^2-x-6}dx = \int 2+\frac{7}{x-3}-\frac{2}{x-2}\,dx$

$= 2x+7\ln|x+2|-2\ln|x+2|+c$

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VRAAG 10

10.1



10.2 $g'(x) = \cos x \tan x + \sin x \sec^2 x = 0$

$\therefore \sin x + \sin x \sec^2 x = 0$

$\therefore \sin x\left(1 + \sec^2 x\right) = 0$

$\therefore \sin x = 0$

$\therefore x = \pi$

$\therefore (\pi;0)$

VRAAG 11

11.1 $P\hat{Q}R = 90°$ ($\angle$ in halfsirkel)

$\therefore \cos\theta = \dfrac{PQ}{4}$

$\therefore PQ = 4\cos\theta$

tyd $= \dfrac{\text{afstand}}{\text{spoed}}$ tyd om te roei $= \dfrac{4}{3}\cos\theta$

Let wel: PQ kan ook bepaal word deur $\perp$ halveerder van koord te gebruik

$Q\hat{O}R = 20$ ($\angle$ by middelpunt)

$\therefore QR = r\theta = 4\theta$

dus tyd om te stap $= \dfrac{4\theta}{5}$

totale tyd $t = \dfrac{4}{3}\cos\theta + \dfrac{4\theta}{5}$

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11.2 $t = \frac{4}{3}\cos\theta + \frac{4\theta}{5}$

$\frac{dt}{d\theta} = -\frac{4}{3}\sin\theta + \frac{4}{5} = 0$

$\therefore -\frac{4}{3}\sin\theta = -\frac{4}{5}$

$\therefore \sin\theta = \frac{3}{5}$

$\therefore \theta = \arcsin\left(\frac{3}{5}\right) = 0,644$

Totaal: 200 punte

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