INTERNATIONAL SECONDARY CERTIFICATE EXAMINATION
NOVEMBER 2022

# FURTHER STUDIES MATHEMATICS (STANDARD): PAPER I

## MARKING GUIDELINES

Time: 2 hours 200 marks

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**These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.**

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## QUESTION 1

1.1 (a) $\ln(x-2)=1$ or $\ln(x-2)=-1$

$\therefore x-2=e$ or $x-2=e^{-1}$

$\therefore x=e+2$ or $x=e^{-1}+2$

$\therefore x=4,72$ or $x=2,37$

(b) $\therefore e^x\left(e-1\right)=12$

$$\therefore e^x=\frac{12}{e-1}$$

$$\therefore x=\ln\frac{12}{e-1}=1.94$$

(c) $\therefore -2\left|2x+4\right|\geq -16$

$\therefore \left|2x+4\right|\leq 8$

$\therefore -8\leq 2x+4\leq 8$

$\therefore -12\leq 2x\leq 4$

$\therefore -6\leq x\leq 2$

1.2 $$\frac{15-5ai}{a+2i}\times\frac{a-2i}{a-2i}=\frac{15a-30i-5a^2i+10ai^2}{a^2-4i^2}=-1-7i$$

$$\therefore \frac{5a}{a^2+4}+\frac{-30-5a^2}{a^2+4}i=-1-7i$$

$$\therefore \frac{5a}{a^2+4}=-1$$

$\therefore 5a=-a^2-4$

$\therefore a^2+5a+4=0$

$\therefore \left(a+1\right)\left(a+4\right)=0$

$\therefore a=-1$ or $a=-4$

but, a check reveals that $a=-1$ is the only option which generates the correct imaginary part

$\therefore a=-1$

**ALTERNATE SOLUTION 1**

$\therefore 15-5ai=(-1-7i)(a+2i)$

$\therefore 15-5ai=-a-7ai-2i-14i^2$

$\therefore 15-5ai=13-(7a+2)i$

$\therefore 5a=7a+2$

$\therefore -2a=2$

$\therefore a=-1$

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**ALTERNATE SOLUTION 2**

$$\therefore 15-5ai=(-1-7i)(a+2i)$$
$$15=-a-14i^2$$
$$\therefore a=-1$$

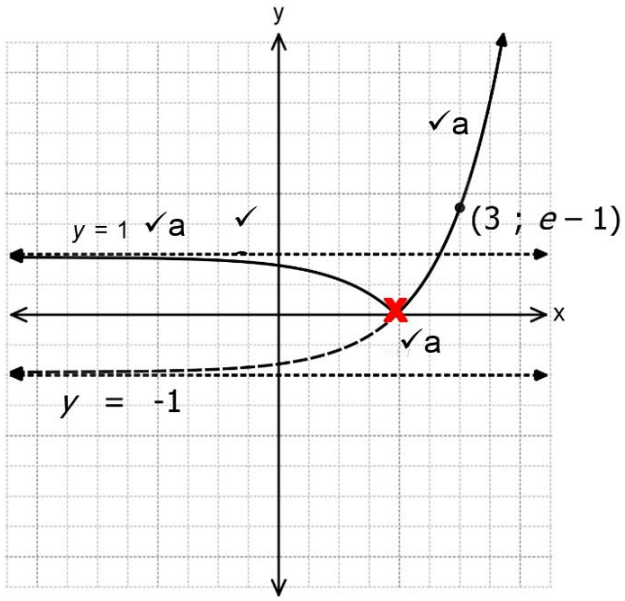
1.3 (a) if $2+i$ is a root then so is $2-i$
sum of roots is 4 and product of roots is 5
$\therefore f(x)=\left(x^2-4x+5\right)$ is a factor
$\therefore f(x)=\left(x^2-4x+5\right)\left(x^2+x-20\right)$
$\therefore f(x)=\left(x^2-4x+5\right)(x+5)(x-4)$

(b) $\left(x^2-4x+5\right)(x+5)(x-4)=0$
$\therefore x=2+i$ or $2-i$ or $-5$ or $4$

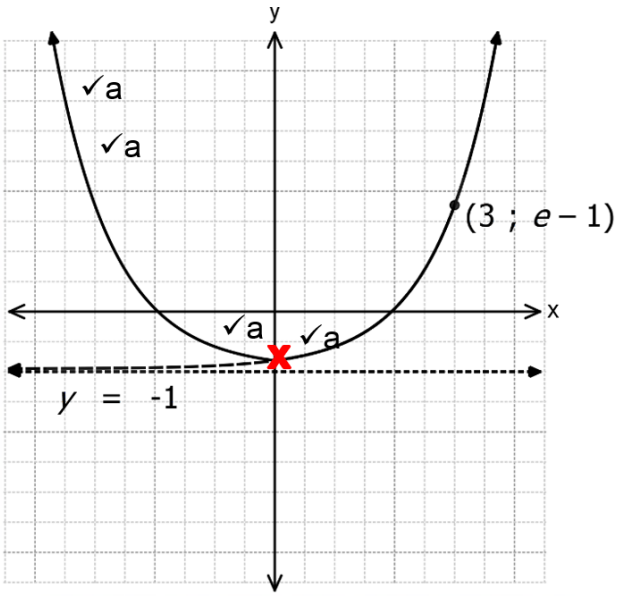
**QUESTION 2**

2.1 (a) $c=-1$
$e-1=e^{3+p}-1$
$\therefore e=e^{3+p}$
$\therefore p=-2$

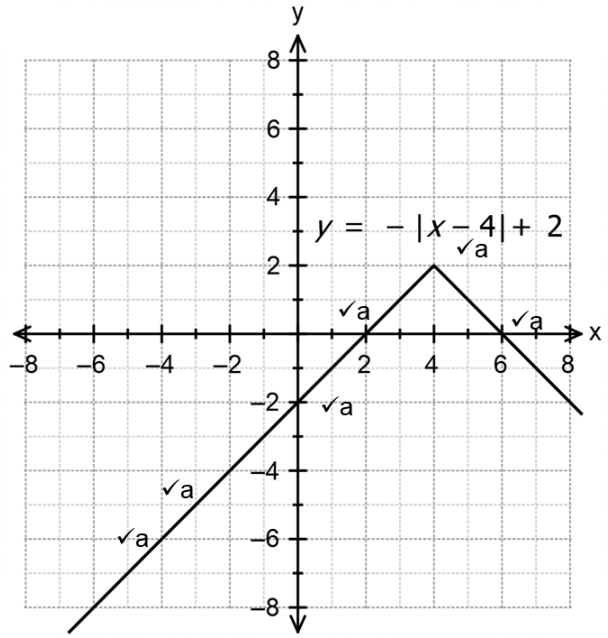
(b) (i)



(ii)




2.2




## QUESTION 3

3.1 $ax - 8 = 0$ when $x = 2$

$\therefore a = 4$

now $2x^2 + bx - 10 = (4x - 8)\left(\frac{1}{2}x + 4\right) + R$

$\therefore 2x^2 + bx - 10 = 2x^2 + 12x - 32 + R$

$\therefore b = 12$

3.2
- A $y$-intercept of 3
- $x$-intercepts of –2 and 3
- Two vertical asymptotes, one of which must be $x = 1$
- A horizontal asymptote of $y = -2$ – y-intercept of 3

$$y = \frac{-2(x+2)(x-3)}{(x-1)(x+4)}$$

3.3 $$g'(x) = \frac{(2x+3)(3x-13) - 3(x^2+3x-10)}{(3x-13)^2} = 0 - \text{quotient rule}$$

$$\therefore 3x^2 - 26x - 9 = 0$$

$$\therefore (3x+1)(x-9) = 0$$

$$\therefore x = -\frac{1}{3} \text{ or } 9$$

$$\therefore \left(-\frac{1}{3}; 0{,}78\right) \text{ and } (9; 7)$$

## QUESTION 4

4.1 (a) $p = 2$ (2)

(b) we need f to be continuous at $x = -2$

now $\lim\limits_{x \to -2^+} f(x) = 1$

$$\therefore \lim_{x \to -2^-} f(x) = 1$$

$$\therefore -2m + c = 1$$

also, $\lim\limits_{x \to -2^+} f'(x) = \lim\limits_{x \to -2^+} (-2x) = 4$

$$\therefore \lim_{x \to -2^-} f'(x) = 4$$

$$\therefore m = 4$$

$$\therefore c = 9$$

4.2 (a) $$x^2 + 10y^2 + 14x + 16y = 2$$

$$\therefore 2x + 20y\frac{dy}{dx} + 14 + 16\frac{dy}{dx} = 0$$

$$\therefore \frac{dy}{dx}(20y + 16) = -2x - 14$$

$$\therefore \frac{dy}{dx} = \frac{-2x - 14}{20y + 16}$$

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(b) $\frac{dy}{dx} = \frac{-2x-14}{20y+16}$

$\left.\frac{dy}{dx}\right|_{(-2;1)} = \frac{-10}{36} = -\frac{5}{18}$

$\therefore$ gradient of normal $= \frac{18}{5}$

$\therefore y - 1 = \frac{18}{5}(x+2)$

$\therefore y = \frac{18}{5}x + \frac{41}{5}$ (this line need not be shown)

## QUESTION 5

5.1 (a) $f(x) = x^3 - 2x + 2 = 0$

$\therefore f'(x) = 3x^2 - 2$

$\therefore x_{n+1} = x_n - \frac{x_n{}^3 - 2x_n + 2}{3x_n{}^2 - 2}$

(b) The tangent at $x - 1$intersects the x-axis at 0 and the tangent at $x = 0$ intersects the x-axis at 1 so the answers cycle rather than converging.

Note that we will accept any comment suggesting 'cycling'/looping.

(c) $x = -1{,}76929$

5.2 (a) $\cos\theta = \frac{3^2 + 4^2 - 6^2}{2(3)(4)} = -\frac{11}{24}$

$\therefore \theta = 2{,}047$

(b) shaded area = area of sector − area of triangle

$= \frac{5^2(2{,}047)}{2} - \frac{1}{2}(3)(4)\sin 2{,}047$

$= 20{,}25\ \text{m}^2$

20,26 if you use the rounded 2,047

## QUESTION 6

$\sum_{i=1}^{n}(3i-1)(3i+2) = 3n^3 + 6n^2 + n$

if $n = 1$ then $LHS = 10$ and $RHS = 10$

so it is true for $n = 1 -$ considering $n = k + 1$

Assume true for $n = k$ viz.

$(2)(5) + (5)(8) + (8)(11) + ... + (3k-1)(3k+2) = k^3 + 6k^2 + k\,(*)$

now if $n = k + 1$ then:

$$(2)(5) + (5)(8) + (8)(11) + ... + (3k-1)(3k+2) + (3k+2)(3k+5) = 3k^3 + 6k^2 + k + (3k+2)(3k+5)$$
$$= 3k^3 + 6k^2 + k + 9k^2 + 21k + 10$$
$$= 3k^3 + 9k^2 + 9k + 3 + 6k + 12k + 6 + k + 1$$

but this is just $(*)$ with $k + 1$ for $k$

so, it is true for $n = k + 1$

$\therefore$ by the principle of mathematical induction it is true for $n \in \mathbb{N}$

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## QUESTION 7

7.1 $10\,200 = 10\,000e^{k}$

$$\therefore e^{k} = \frac{10\,200}{10\,000}$$

$$\therefore k = \ln\frac{102}{100} = 0.0198$$

7.2 $y = y_0 e^{kt}$

$$\therefore \frac{y}{y_0} = e^{kt}$$

$$\therefore kt = \ln\frac{y}{y_0}$$

$$\therefore t = \frac{\ln\frac{y}{y_0}}{k}$$

$$t = \frac{\ln 10}{0{,}0198} = 116{,}29$$

$$\therefore t = 117 \text{ months}$$

## QUESTION 8

$$\text{shaded area } = \int_1^4 f(x)dx - \int_1^4 g(x)dx$$

$$\therefore \frac{21}{2} = \frac{14}{3} - \int_1^4 f(x) - kx - k\,dx$$

$$\therefore \frac{21}{2} = \frac{14}{3} - \left(\int_1^4 f(x)dx - \int_1^4 kx + k\,dx\right)$$

$$\therefore \frac{21}{2} = \frac{14}{3} - \frac{14}{3} + \left[\frac{kx^2}{2} + kx\right]_1^4$$

$$\therefore \frac{21}{2} = 12k - \left(\frac{k}{2} + 1\right)$$

$$\therefore 21 = 24k - k - 2$$

$$23 = 23k$$

$$\therefore k = 1$$

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## ALTERNATE METHOD

$$\frac{21}{2} = \int_1^4 f(x) - g(x)\ dx$$
$$\frac{21}{2} = \int_1^4 f(x) - f(x) + kx + k\ dx$$
$$\frac{21}{2} = \int_1^4 kx + k\ dx$$
$$\frac{21}{2} = \left[\frac{kx^2}{2} + kx\right]_1^4$$
$$\frac{21}{2} = 8k + 4k - \frac{k}{2} - k$$
$$21 = 16k + 8k - k - 2k$$
$$k = 1$$

## QUESTION 9

9.1
$$\int -\text{cosec}^2\theta \cot\theta\, d\theta$$
$$= \int \text{cosec}\theta\left(-\text{cosec}\theta\cot\theta\right)d\theta$$
$$= \frac{\text{cosec}^2\theta}{2} + c$$

9.2
$$\int \frac{3x}{\sqrt{2x+5}\ \checkmark\text{a}}\,dx$$
$$= 3\int x\left(2x^2+5\right)^{-\frac{1}{2}}dx$$
$$= \frac{3\left(2x^2+5\right)^{\frac{1}{2}}}{\frac{1}{2}\times 4} + c$$
$$= \frac{3\left(2x^2+5\right)^{\frac{1}{2}}}{2} + c$$

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**ALTERNATE**

$$= 3\int x\left(2x^2+5\right)^{-\frac{1}{2}}dx$$

let $u = 2x^2+5$ then $\frac{du}{dx} = 4x$ so $dx = \frac{du}{4xdx}$

$$= \frac{3}{4}\int u^{-\frac{1}{2}}du$$

$$= \frac{3}{4}\left(\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right)+c$$

$$= \frac{3u^{\frac{1}{2}}}{2}+c$$

$$= \frac{3\left(2x^2+5\right)^{\frac{1}{2}}}{2}+c$$

9.3 $\frac{2x^2+3x+8}{x^2-x-6}$

$$= \frac{2\left(x^2-x-6\right)+5x+20}{x^2-x-6}$$

$$= \frac{2\left(x^2-x-6\right)}{x^2-x-6}+\frac{5x+20}{x^2-a-6}$$

$$= 2+\frac{5x+20}{(x-3)(x+2)}$$

now $\frac{5x+20}{(x-3)(x+2)} = \frac{A}{x-3}+\frac{B}{x+2}$

$A = 7$ and $B = -2$ both by cover-up method

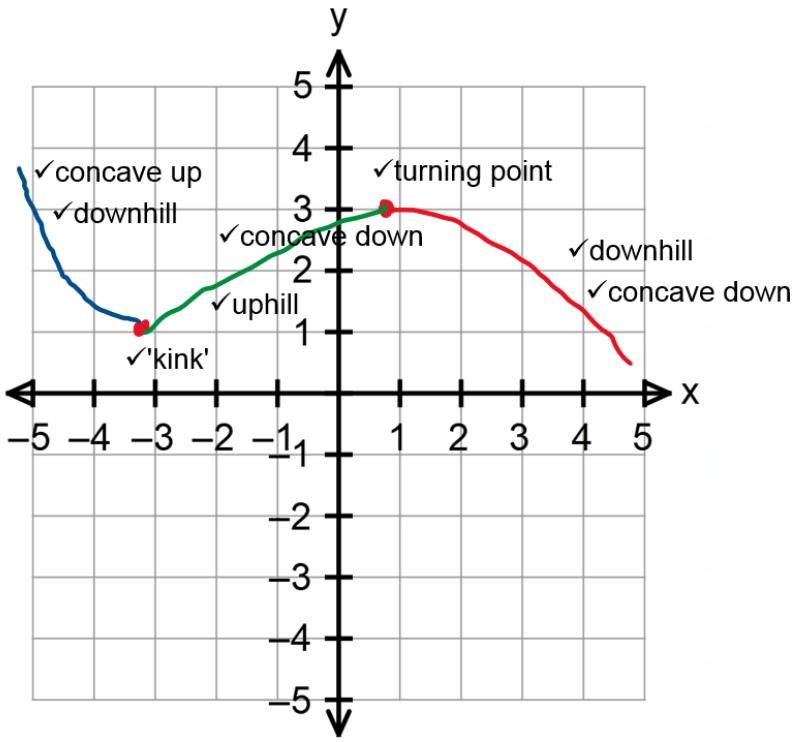
$$\therefore \int \frac{2x^2+3x+8}{x^2-x-6}dx = \int 2+\frac{7}{x-3}-\frac{2}{x-2}\,dx$$

$$= 2x+7\ln|x+2|-2\ln|x+2|+c$$

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## QUESTION 10

10.1




10.2 $g'(x) = \cos x \tan x + \sin x \sec^2 x = 0$

$\therefore \sin x + \sin x \sec^2 x = 0$

$\therefore \sin x\left(1 + \sec^2 x\right) = 0$

$\therefore \sin x = 0$

$\therefore x = \pi$

$\therefore (\pi; 0)$

## QUESTION 11

11.1 $P\hat{Q}R = 90°$ $(\angle$ ub semi-circle$)$

$\therefore \cos\theta = \dfrac{PQ}{4}$

$\therefore PQ = 4\cos\theta$

time $= \dfrac{\text{distance}}{\text{speed}}$ time to row $= \dfrac{4}{3}\cos\theta$

Note: $PQ$ can also be established using $\perp$ bisector of chord

$Q\hat{O}R = 20$ $(\angle$ at centre$)$

$\therefore QR = r\theta = 4\theta$

so, time to walk $= \dfrac{4\theta}{5}$

total time $t = \dfrac{4}{3}\cos\theta + \dfrac{4\theta}{5}$

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11.2 $t = \frac{4}{3}\cos\theta + \frac{4\theta}{5}$

$\frac{dt}{d\theta} = -\frac{4}{3}\sin\theta + \frac{4}{5} = 0$

$\therefore -\frac{4}{3}\sin\theta = -\frac{4}{5}$

$\therefore \sin\theta = \frac{3}{5}$

$\therefore \theta = \arcsin\left(\frac{3}{5}\right) = 0,644$

**Total: 200 marks**

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