NASIONALE SENIOR SERTIFIKAAT-EKSAMEN
MEI 2021

WISKUNDE: VRAESTEL I

NASIENRIGLYNE

Tyd: 3 uur 150 punte

Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en hulp-eksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.

Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.

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LET WEL:

- Indien 'n leerder 'n vraag meer as een keer beantwoord, sien slegs die EERSTE poging na.
- Deurlopende akkuraatheid geld vir alle aspekte van die nasienmemorandum.

AFDELING A

VRAAG 1

(a) (1)	$f(1) = -3(1)^2 + 1$ $f(1) = -2$ $f(2) = -3(2)^2 + 1$ $f(2) = -11$ $m = \frac{-11-(-2)}{2-1}$ $\therefore m = -9$	$f(1) = -2$ $f(2) = -11$ $\therefore m = -9$
(2)	$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ $f'(x) = \lim_{h\to 0} \frac{-3(x+h)^2 + 1 - (-3x^2+1)}{h}$ $f'(x) = \lim_{h\to 0} \frac{-3(x^2+2xh+h^2)+1+3x^2-1}{h}$ $f'(x) = \lim_{h\to 0} \frac{-3x^2-6xh-3h^2+1+3x^2-1}{h}$ $f'(x) = \lim_{h\to 0} \frac{h(-6x-3h)}{h}$ $f'(x) = \lim_{h\to 0}(-6x-3h)$ $= -6x$ **OF** $f(x+h) = -3(x+h)^2 + 1$ $f(x+h) = -3(x^2+2xh+h^2)+1$ $f(x+h) = -3x^2-6xh-3h^2+1$ $f(x+h) - f(x) = -3x^2-6xh-3h^2+1+3x^2-1$ $f(x+h) - f(x) = -6xh-3h^2$ $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ $f'(x) = \lim_{h\to 0} \frac{h(-6x-3h)}{h}$ $f'(x) = \lim_{h\to 0}(-6x-3h)$ $= -6x$	$-3(x+h)^2 + 1 - (-3x^2+1)$ Vereenvoudiging Faktorisering $\lim_{h\to 0}(-6x-3h)$ $= -6x$ $-3(x+h)^2 + 1 - (-3x^2+1)$ Vereenvoudiging Faktorisering $\lim_{h\to 0}(-6x-3h)$ $= -6x$

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(b) (1)	$g(x)=2x^2+2x^{-1}$ $g'(x)=4x-2x^{-2}$ $g'(x)=4x-\frac{2}{x^2}$	$4x$ $-2x^{-2}$
(2)	$g\left(\frac{1}{x}\right)=2\left(\frac{1}{x}\right)^2+\frac{2}{\frac{1}{x}}$ $g\left(\frac{1}{x}\right)=\frac{2}{x^2}+2x$	korrekte vervanging $\frac{2}{x^2}+2x$
(3)	$g'(x)+g\left(\frac{1}{x}\right)$ $=4x-\frac{2}{x^2}+\frac{2}{x^2}+2x$ $=6x$	vereenvoudiging $6x$
(c) (1)	$f'(x)=6x^2-10x+1$ $f'(x)=6(2)^2-10(2)+1$ $f'(2)=5$	$6x^2-10x+1$ vervanging $f'(2)=5$
(2)	$f(2)=-3$ Vergelyking van raaklyn: $y=5x+c$ Vervanging: $(2;-3)$ $c=-13$ $\therefore y=5x-13$	$f(2)=-3$ $c=-13$

VRAAG 2

(a)	Antwoord: Opsie 4	antwoord
(b)	$\frac{dA}{dx}=30-4x$ $30-4x=0$ $x=7\frac{1}{2}$ Dus is die afmetings: $7\frac{1}{2}$ by 15	$30-4x$ $30-4x=0$ antwoord

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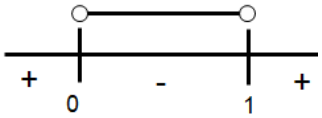
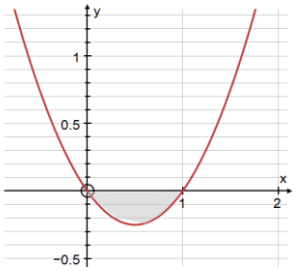
VRAAG 3

(a)	$x=-\frac{1}{3}$ of $x=\frac{5}{3}$ $\left(x+\frac{1}{3}\right)\left(x-\frac{5}{3}\right)=0$ $x^2-\frac{4}{3}x-\frac{5}{9}=0$ KGV: 9 $9x^2-12x-5=0$ $\therefore b=-12$ OF $(3x+1)(3x-5)=0$ $9x^2-12x-5=0$ $\therefore b=-12$	$\left(x+\frac{1}{3}\right)\left(x-\frac{5}{3}\right)=0$ $9x^2-12x-5=0$ $\therefore b=-12$ $(3x+1)(3x-5)=0$ $9x^2-12x-5=0$ $\therefore b=-12$
(b) (1)	$\sqrt{8x+12}=x-1$ $8x+12=x^2-2x+1$ $x^2-10x-11=0$ $(x-11)(x+1)=0$ x = 11 of x = –1 Kontroleer: $x=-1$ is nie geldig nie	Isoleer wortelvorm x^2-2x+1 $x^2-10x-11$ faktore antwoord met seleksie
(2)	$x^2-x<0$ $x(x-1)<0$ Kritieke waardes: 0 ; 1  Oplossing: $0<x<1$	 Faktore/kritieke waardes Getallelyn/grafiek $0<x<1$

(c)	$\left(2^4\right)^{\frac{p}{4}} = \frac{2^{4x+1}}{2^{3x}}$ $2^p = 2^{x+1}$ $x = p-1$	2^p 2^{x+1} antwoord
(d)	$x^2 + px - 1 = 0$ $x = \frac{-p \pm \sqrt{p^2+4}}{2}$ Vir die wortels om gelyk te wees, $p^2 + 4 = 0$ $p^2 = -4$ Dus geen reële oplossing nie **OF** $\Delta = 0$ vir gelyke wortels $p^2 + 4 = 0$ $p^2 = -4$ Dus geen reële oplossing nie	$\frac{-p \pm \sqrt{p^2+4}}{2}$ $p^2 = -4$ Dus geen reële oplossing nie $\Delta = 0$ vir gelyke wortels $p^2 = -4$ Dus geen reële oplossing nie

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VRAAG 4

(a) (1)	$F = 12\,895\left[\dfrac{\left(1+\dfrac{13}{200}\right)^{3\times 2}-1}{\dfrac{13}{200}}\right]$ $F =$ R91086,76791	Vervanging in korrekte formule akkurate vervanging antwoord
(2)	$345\,000 = 12\,895\left[\dfrac{\left(1+\dfrac{13}{200}\right)^{2n}-1}{\dfrac{13}{200}}\right]$ $2,739\,046\,142 = (1{,}065)^{2n}$ $\log_{1{,}065} 2,739\,046\,142 = 2n$ $n \approx 8$ jaar	Vervanging in korrekte formule $2n$ vereenvoudiging omskakeling in logvorm antwoord
(3)	$A = 345\,000\left(1+\dfrac{3{,}5}{100}\right)^{8}$ $A =$ R454 299,1178	Vervanging in korrekte formule Akkurate vervanging antwoord
(b)	$P = \dfrac{x\left[1-(1+i)^{-n}\right]}{i}$ $345\,000 = \dfrac{x\left[1-\left(1+\dfrac{13}{200}\right)^{-8\times 2}\right]}{\dfrac{13}{200}}$ $x =$ R 35 320,26301	Vervanging in korrekte formule Akkurate vervanging antwoord

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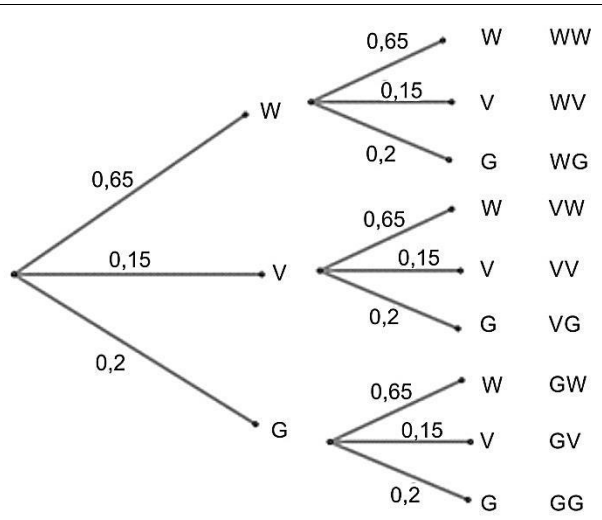
VRAAG 5

(a) (1)		Boomdiagram wat 2 wedstryde voorstel korrekte waarskynlikhede aangedui korrekte etikette
(2)	P(wen & gelykop in enige volgorde) = P(WG) + P(GW) $=(0,65\times0,2)+(0,2\times0,65)$ $=\frac{13}{50}$ $=0,26$	 $(0,65\times0,2)$ $(0,2\times0,65)$ $=0,26$
(3)	Hy is nie korrek nie, want die waarskynlikheid dat ’n gebeurtenis sal plaasvind, kan nie groter wees as 1 nie.	regverdiging
(b) (1)	10! $=3\,628\,800$	10! $=3\,628\,800$
(2)	$10\times9\times8\times7$ $=5\,040$ **OF** $\frac{10!}{(10-4)!}$ $=5\,040$	$10\times9\times8\times7$ $=5\,040$ 10! $(10-4)!$ $=5\,040$
(c)	5! × 4! $=2\,880$	5! 4! $=2\,880$

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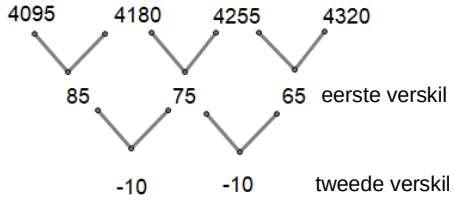
AFDELING B

VRAAG 6

(a)	$S_n = \frac{n}{2}(a+l)$ $S_{900} = \frac{900}{2}(1+900)$ $S_{900} = 405\,450$ Getal terme wat veelvoude van 5 is: 180 $S_{180} = \frac{180}{2}(5+900)$ $S_{180} = 81\,450$ Som van oorblywende getalle $= 405\,450 - 81\,450$ $= 324\,000$	Korrekte formule $300 = \frac{n}{2}(3+47)$ $S_{900} = 405\,450$ 180 $(5+900)$ $S_{180} = 81\,450$ 324 000
(b)	$S_n = \frac{a(r^n - 1)}{r-1}$ $a = 49$ en $d = \frac{6}{7}$ $300 = \frac{49\left(\left(\frac{6}{7}\right)^n - 1\right)}{\frac{6}{7} - 1}$ $\left(\frac{6}{7}\right)^n = \frac{43}{343}$ $\log_{\left(\frac{6}{7}\right)}\left(\frac{43}{343}\right) = n$ $n \approx 13{,}47$ $\therefore n = 14$	$d = \frac{6}{7}$ Vervanging in korrekte formule vereenvoudiging $\log_{\left(\frac{6}{7}\right)}\left(\frac{138}{343}\right) = n$ antwoord
(c)	$r = t - 5$ $-1 < t - 5 < 1$ $4 < t < 6$	$r = t - 5$ $-1 < \overset{r}{x} < 1$ $t > 4$ $t < 6$

(d) (1)	$n=1$: $T_n=[100+5(1)]\times(40-1)$ $=4\,095$ $n=2$: $T_2=4\,180$ $T_3=4\,255$ $T_4=4\,320$  	4 095, 4 180, 4 255, 4 320 eersteverskil-terme tweede verskil
(2)	$T_n=(100+5n)(40-n)$ $T_n=-5n^2+100n+4\,000$ Wanneer $n=40$ sal totale koste nul wees. **OF** $2a=-10$ $3a+b=85$ $a+b+c=4\,095$ $a=-5$ $b=100$ $c=4\,000$ $T_n=-5n^2+100n+4\,000$ Wanneer $n=40$ sal totale koste nul wees. **OF** $2a=-10 \quad \therefore a=-5$ $T_n=-5n^2+bn+c$ $T_1:-5(1)^2+b(1)+c=4\,095 \quad \therefore \quad b+c=4\,100$ $T_2:-5(2)^2+b(2)+c=4\,180 \quad \therefore \quad 2b+c=4\,200$ $T_2-T_1:b=100$ $\therefore c=4\,000$ $T_n=-5n^2+100n+4\,000$ Wanneer $n=40$ sal totale koste nul wees.	$a=-5$ $b=100$ $c=4000$ $n=40$, koste = 0 $a=-5$ $b=100$ $c=4000$ $n=40$, koste = 0 $a=-5$ $b=100$ $c=4000$ $n=40$, koste = 0

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VRAAG 7

(a)	$\frac{1}{2}x^2 = -\frac{1}{x+1}+1$ KGV: $2(x+1)$ Beperking: $x \neq -1$ $x^2(x+1) = -1(2) + 2(x+1)$ $x^3 + x^2 = -2 + 2x + 2$ $x^3 + x^2 - 2x = 0$ $x(x^2 + x - 2) = 0$ $x(x+2)(x-1) = 0$ $x = 0$ of $x = -2$ of $x = 1$ $\therefore A(-2;2)$ en $B\left(1;\frac{1}{2}\right)$	Stel gelyk vir snypunte vereenvoudiging $x^3 + x^2 - 2x$ faktorisering $x(x+2)(x-1)$ $x = 0$ of $x = -2$ of $x = 1$ $A(-2;2)$ $B\left(1;\frac{1}{2}\right)$
(b)	$g(x) > f(x)$ vir: $-2 < x < -1$ of $0 < x < 1$	$-2 < x < -1$ $0 < x < 1$ $x < 1$
(c)	$g: y = -\frac{1}{x+1}+1$ $g^{-1}: x = -\frac{1}{y+1}+1$ $\frac{1}{y+1} = -x+1$ $(y+1)(-x+1) = 1$ $y(-x+1) = 1-1+x$ $y = \frac{-(-x+1)+1}{(-x+1)}$ $y = -\frac{1}{x-1}-1$	Verander x en y $(y+1)(-x+1) = 1$ $y = \frac{-(-x+1)+1}{(-x+1)}$ $y = -\frac{1}{x-1}-1$

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VRAAG 8

(a)	Funksie daal wanneer: $g'(x) < 0$ $g'(x) = 9x^2 - 4k^2$ $9x^2 - 4k^2 < 0$ $\therefore (3x - 2k)(3x + 2k) < 0$ Kritieke waardes: $\pm \frac{2k}{3}$ $-\frac{2k}{3} < x < \frac{2k}{3}$	$g'(x) < 0$ $9x^2 - 4k^2$ $\therefore (3x - 2k)(3x + 2k) < 0$ Kritieke waardes: $\pm \frac{2k}{3}$ antwoord
(b) (1)	$g(x) = \frac{a}{x-1} + q$ vervang $(-2;0)$ $0 = \frac{a}{-2-1} + q$ $q = \frac{a}{3}$ ... vergelyking 1 $g(x) = \frac{a}{x-1} + q$ vervang $(0;-2)$ $-2 = \frac{a}{0-1} + q$ $-2 = -a + q$ ... vergelyking 2 Vervang vergelyking 1 in vergelyking 2 $-2 = -a + \frac{a}{3}$ $a = 3$ $\therefore q = 1$	vervang $(-2;0)$ $q = \frac{a}{3}$ vervang $(0;-2)$ $-2 = -a + q$ $a = 3$ $q = 1$
(2)	$f(x) = x^3 - 3x + t$ $f'(x) = 3x^2 - 3$ $f''(x) = 6x$ Vir konkaaf afwaarts: $f''(x) < 0$ $6x < 0$ $x < 0$	$f'(x) = 3x^2 - 3$ $f''(x) < 0$ $x < 0$
(3)	$f(x) = x^3 - 3x - 2$ $f(x) - k = 0$ het 3 reële wortels vir: $-4 < k < 0$	$f(x) = x^3 - 3x - 2$ $-4 < k < 0$

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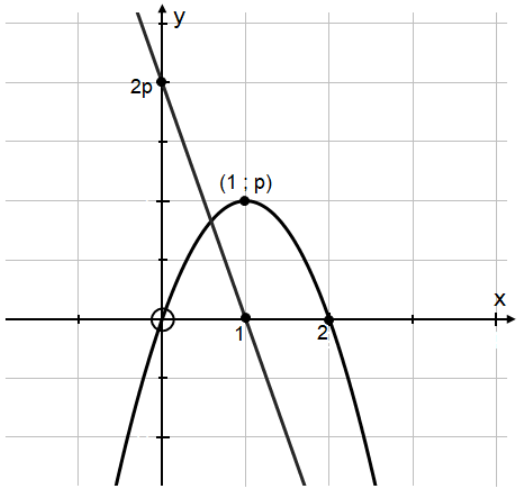
VRAAG 9

(a) (1)		maksimumwaarde-parabool Draaipunt (1 ; p) X-afsnit: (0;0) X-afsnit: (2;0) Y-afsnit: (0;0) reguitlyn met negatiewe gradiënt X-afsnit: (1;0) Y-afsnit: (0;2p)
(2)	$f(x)=f'(x)$ $-px^2+2px=-2px+2p$ $-px^2+4px-2p=0$ $-p(x^2-4x+2)=0$ $\therefore$ Vir snypunt: $(x^2-4x+2)=0$ $\therefore$ Onafhanklik van p OF $-px^2+2px=-2px+2p$ $-px^2+4px-2p=0$ $-p(x^2-4x+2)=0$ $\therefore$ Vir snypunt by: $x=2\pm\sqrt{2}$ $\therefore$ Onafhanklik van p	$-px^2+2px=-2px+2p$ $-p(x^2-4x+2)=0$ $(x^2-4x+2)=0$ $-px^2+2px=-2px+2p$ $-p(x^2-4x+2)=0$ $x=2\pm\sqrt{2}$
(b) (1)	$g(x)=-x+1$ $\therefore$ y-afsnit: $(0;1)$ $\therefore P(0;1)$ Vir Q : vervang $(x;1)$ in $f(x)=\log_3 x$ $1=\log_3 x$ $x=3$ $\therefore Q(3;1)$ Vir R : vervang $(3;y)$ in $g(x)=-x+1$ $y=-3+1$ $y=-2$ $\therefore R(3;-2)$	$\therefore P(0;1)$ $1=\log_3 x$ $\therefore Q(3;1)$ $\therefore R(3;-2)$

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	Vir S: vervang $(x;-2)$ in $f(x)=\log_3 x$ $-2=\log_3 x$ $3^{-2}=x$ $x=\frac{1}{9}$ $\therefore S\left(\frac{1}{9};-2\right)$	$3^{-2}=x$ $\therefore S\left(\frac{1}{9};-2\right)$
(2)	$k(x)=a.3^x-2$ Vervang: x-afsnit $(0;-1)$ $-1=a.b^0-2$ $a=1$ Alternatief: As $a<0$ $q=-2$ maar y-afsnit $(0;-3)$ $y=(3)^x-2$ $-3=a(3)^0-2$ vervang$(0;-3)$ $-1=a$	$q=-2$ $(0;-1)$ $-1=a.b^0-2$ $a=1$

VRAAG 10

V 10	Afstand = Spoed $\times$ tyd Laat John se spoed = x Simon se spoed = $x+4$ Simon: $150=(x+4)(t)$ ...vergelyking 1 John: $150=x(t+0,5)$ ...vergelyking 2 $\therefore$ vergelyking 1 = vergelyking 2 $tx+4t=tx+0,5x$ $4t=0,5x$ $x=8t$ …. Vervang in vergelyking 1 $(8t+4)(t)=150$ $8t^2+4t-150=0$ $t\approx 4,09$ of $t\approx -4,59$ Nie geldig nie John se spoed: $150=x(4,09+0,5)$ $150=4,59x$ $x=32,7$ km/h OF $\frac{150}{x+4}+\frac{1}{2}=\frac{150}{x}$ KGV: $2x(x+4)$ $150(2x)+x(x+4)=150(2)(x+4)$ $150(2x)+x(x+4)=150(2)(x+4)$ $\therefore x\approx 32,7$ km/h	korrekte formule $150=(x+4)(t)$ $150=x(t+0,5)$ $tx+4t=tx+0,5x$ $t\approx 4,09$ $x=32,7$ $\frac{150}{x+4}+\frac{1}{2}=\frac{150}{x}$ $2x(x+4)$ $150(2x)+x(x+4)=150(2)(x+4)$ $150(2x)+x(x+4)=150(2)(x+4)$ $\therefore x\approx 32,7$

Totaal: 150 punte

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