NASIONALE SENIOR SERTIFIKAAT-EKSAMEN
NOVEMBER 2020

# TEGNIESE WISKUNDE: VRAESTEL I

## NASIENRIGLYNE

Tyd: 3 uur 150 punte

---

**Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en hulp-eksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.**

**Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.**

---

IEB Copyright © 2020

## VRAAG 1

1.1 1.1.1
$$2x^2 - 5x = 12$$
$$2x^2 - 5x - 12 = 0 \ \checkmark$$
$$(2x+3)(x-4) = 0 \ \checkmark$$
$$x = -\frac{3}{2} \text{ of } x = 4 \ \checkmark \quad (3)$$

1.1.2
$$4x + 7 = -2x^2$$
$$2x^2 + 4x + 7 = 0$$
$$x^2 + 2x + \frac{7}{2} = 0$$

**OF**

**Opsie 1**

Gebruik formule
$$x = \frac{-4 \pm \sqrt{16 - 56}}{2} \ \checkmark\checkmark$$
$$= \frac{-4 \pm \sqrt{-40}}{2} \ \checkmark$$
$$= \frac{-2 \pm i\sqrt{10}}{2} \ \checkmark \text{ (gebruik berekening)}$$
$$= -1 \pm \frac{\sqrt{10}}{2}i \ \checkmark$$

**Opsie 2**
$$(x+1)^2 = 1 - \frac{7}{2} = \frac{-5}{2} = \frac{-10}{4} \ \checkmark\checkmark$$
$$x + 1 = \pm\frac{\sqrt{10i}}{2} \ \checkmark$$
$$x = -1 \pm \frac{\sqrt{10i}}{2} \ \checkmark \quad (4)$$

1.2 Laat energieverbruik van motor $= x$ en verwarmer $= y$

$4x + 2y = 25$ ① ✓ en $2x + 3y = 18$ ② ✓

② × 2: $4x + 6y = 36$

Trek af $-4y = -11$ ✓

$$y = \frac{11}{4} \text{ kJ/h} \ \checkmark$$

Vervang ① $4x + \frac{11}{2} = 25$
$$4x = \frac{39}{2}$$
$$x = \frac{39}{8} \text{ kJ/h} \ \checkmark \quad (5)$$

1.3
$$\Delta = (-3)^2 - 4(1)(9k) \ \checkmark$$
$$= 9 - 36k \ \checkmark$$

Vir reële, verskillende wortels $\Delta > 0$
$$9 - 36k > 0 \ \checkmark$$
$$-36k > -9$$
$$k < \frac{1}{4} \ \checkmark \quad (4)$$

**[16]**

## VRAAG 2

2.1
$$\frac{3^{2014}+9^{1007}}{27^{671}}$$
$$=\frac{3^{2014}+3^{2014}}{3^{2013}}\ \checkmark\checkmark$$
$$=\frac{2.3^{2014}}{3^{2013}}\ \checkmark$$
$$=2.3=6\ \checkmark$$

**OF**

$$\frac{3^{2014}}{3^{2013}}+\frac{3^{2014}}{2^{2013}}\ \checkmark$$
$$=3+\ \checkmark\ 3=6\ \checkmark$$

(4)

2.2 2.2.1
$$5-\sqrt{4x+1}=x$$
$$5-x=\sqrt{4x+1}\ \checkmark$$
$$25-10x+x^2=4x+1\ \checkmark$$
$$x^2-14x+24=0\ \checkmark$$
$$(x-2)(x-12)=0\ \checkmark$$
$$x=2\ \checkmark \text{ of } x=12 \text{ NVT}\ \checkmark\checkmark$$

**OF** Kontroleer oplossing

$5-x\geq 0$ en $4x+1\geq 0$

$-x\geq -5$ en $x\geq -\frac{1}{4}$

$x\leq 5$

(6)

2.2.2
$$2\log x=\log 4+\log(x-1)$$
$$\log x^2=\log(4x-4)\ \checkmark\checkmark\ x>1$$
$$x^2=4x-4$$
$$x^2-4x+4=0\ \checkmark$$
$$(x-2)^2=0\ \checkmark$$
$$x=2\ \checkmark$$

(5)

**[15]**

IEB Copyright © 2020 

## VRAAG 3

3.1 $\frac{(3-2i)}{(1+5i)}.\frac{(1-5i)}{(1-5i)}$ ✓

$= \frac{3-17i+10i^2}{1-25i^2}$ ✓

$= \frac{3-17i-10}{1+25}$ $(i^2=-1)$ ✓

$= \frac{-7-17i}{26}$ ✓

$= \frac{-7}{26}-\frac{17}{26}i$ ✓ (5)

3.2 3.2.1 $V = 2(\cos 120° + i\sin 120°)$ ✓ ✓ **of** $2(-\cos 60° + i\sin 60°)$ (2)

3.2.2 $V = 2\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)$ ✓✓

$= -1+\sqrt{3}i$ ✓ ✓ (4)

3.3 $11001_2$

$= 2^4+2^3+2^0$ ✓

$= 16+8+1$ ✓

$= 25$ ✓ (3)

**[14]**

IEB Copyright © 2020 

## VRAAG 4

4.1 $= 4\,800 - \left(4\,800 \times \frac{13{,}5}{100}\right)$ ✓

$= \text{R}4\,152$ ✓

**OF** $\text{R}4\,800 \times 86{,}5\%$ ✓

$= \text{R}4\,152$ ✓ (2)

4.2 4.2.1 $m = 4$

$i = 0{,}09$

$$1 + i = \left(1 + \frac{i^{(m)}}{m}\right)^m$$

$$1 + 0{,}09 \text{ ✓} = \left(1 + \frac{i^{(4)}}{4}\right)^4 \text{ ✓}$$

$$\sqrt[4]{1{,}09} \text{ ✓} = 1 + \frac{i^{(4)}}{4}$$

$$\sqrt[4]{1{,}09} - 1 = \frac{i^{(4)}}{4}$$

$$4\left(\sqrt[4]{1{,}09 - 1}\right) = i^{(4)} \text{ ✓}$$

$$\therefore i^{(4)} = 8{,}71\% \text{ ✓}$$

**OF** Die nominale rentekoers is 8,71% p.j. kwartaalliks saamgestel ✓ (5)

4.2.2 Laat bedrag belê $= x$ 2

$$3x = x(1 + 15i) \text{ ✓}$$

$$3 = 1 + 15i \text{ ✓}$$

$$i = \frac{2}{15} = 0{,}133 \text{ ✓}$$

d.w.s. $13\frac{1}{3}\%$ p.j. ✓ (4)

4.3 Oorspronklike waarde $= x$

$$\frac{x}{2} = x(1 - 0{,}13)^n \text{ ✓}$$

$$0{,}5 = 0{,}87^n \text{ ✓}$$

$$\log_{0{,}87} 0{,}5 = n \text{ ✓}$$

$n = 4{,}977\ldots$ d.w.s. 5 jaar ✓ (4)

**[15]**

**VRAAG 5**

5.1 Vervang $(-2;5)$ in $g$: $5 = -(-2) + k$ ✓ ✓

$3 = k$ (2)

5.2 T is $(0;3)$ d.w.s. $c = 3$ ✓ **OF** $y = -\frac{1}{2}(x+2)^2 + 5$ ✓

$f(x) = -\frac{1}{2}x^2 + bx + 3$ ✓ $= -\frac{1}{2}x^2 + 2x - 2 + 5$ ✓

Vervang $(-2;5)$: $5 = -\frac{1}{2}(-2)^2 + b(-2) + 3$ ✓ $= -\frac{1}{2}x^2 - 2x - 3$ ✓

$2b = -4$ $c = 3$; $b = -2$ ✓

$b = -2$ ✓ (4)

5.3 $PQ = y_P - y_Q$

$= (-x + 3) - \left(-\frac{1}{2}x^2 - 2x + 3\right)$ ✓

$= \frac{1}{2}x^2 + x$ ✓ (2)

5.4 $\frac{1}{2}x^2 + x = 12$ ✓

$x^2 + 2x - 24 = 0$

$(x+6)(x-4) = 0$ ✓

$x = -6$ of $x = 4$

P ✓ NVT ✓

Vervang in $g$: $y_P = -(-6) + 3 = 9$ ✓

P is $(-6;9)$ (5)

5.5 $g(x) \leq f(x)$

✓ ✓ ✓

$-2 \leq x \leq 0$ ✓ **OF** $[-2;0]$ (2)

**[15]**

IEB Copyright © 2020 

## VRAAG 6

6.1 $y = 1$ ✓ (1)

6.2 By B, $x = 0$: $y = 2^\circ + 1$ ✓

$= 1 + 1 = 2$ ✓

d.w.s. $r = 2$ dus $g(x) = \sqrt{4 - x^2}$ ✓ (3)

6.3 Definisiegebied: $x \in [-2\,;2]$ ✓ **OF** $-2 \le x \le 2$

Waardegebied: $y \in [0\,;2]$ ✓ $0 \le y \le 2$ (2)

6.4 $M_{AB} = \dfrac{y_A - y_B}{x_A - x_B}$

✓

$0,44 = \dfrac{k-2}{-1,466-0}$ ✓

$0,44(-1,466) = k - 2$ ✓

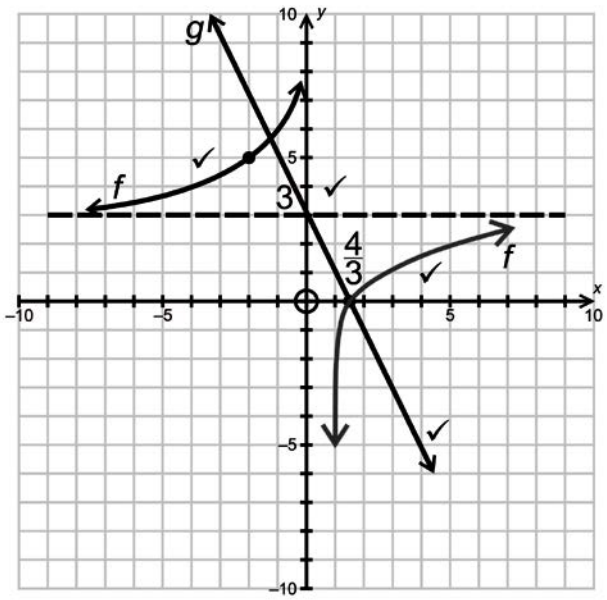
$k \approx 1,35$ ✓ (4)

**[10]**

IEB Copyright © 2020 

## VRAAG 7

7.1




'n Asimptoot $y = 3$

Laat $y = 0$ $\quad 0 = -\frac{4}{3} + 3 \quad \Rightarrow x = \frac{4}{3}$

Punte op grafiek toegeken. (4)

7.2 In $g = \frac{3-0}{0-4} = -\frac{9}{4}$ ✓

Vergelyking van $g : y = -\frac{9}{4}x + 3$ ✓

$$\therefore h(x) = -\frac{9}{4}x + 3 + 1$$

$$= -\frac{9}{4}x + 4 \text{ ✓}$$ (3)

**[7]**

## VRAAG 8

8.1
$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \checkmark$$
$$= \lim_{h\to 0} \frac{5-2x-2h-5+2x}{h} \checkmark$$
$$= \lim_{h\to 0} \frac{-2h}{h} \checkmark$$
$$= \lim_{h\to 0} -2 \checkmark$$
$$= -2 \checkmark$$ (5)

8.2
$$y = \frac{x^2}{x} - \frac{4x}{x} + \frac{3}{x} \checkmark$$
$$= x - 4 + 3x^{-1} \checkmark$$
$$\therefore \frac{dy}{dx} = 1 - 3x^{-2} \checkmark\checkmark$$

**OF**

$$= 1 - \frac{3}{x^2}$$ (4)

8.3
$$f(x) = 2x^{\frac{1}{2}} + x^{-3} - \sqrt{2}x \checkmark$$
$$f'(x) = x^{-\frac{1}{2}} \checkmark - 3x^{-4} \checkmark - \sqrt{2} \checkmark$$

**OF**

$$= \frac{1}{\sqrt{x}} - \frac{3}{x^4} - \sqrt{2}$$ (4)

**[13]**

IEB Copyright © 2020 

## VRAAG 9

9.1 9.1.1 (a)

$$\text{Vol} = \pi r^2 h$$
$$375 = \pi r^2 h \ \checkmark$$
$$\frac{375}{\pi r^2} = h \ \checkmark \qquad (2)$$

(b)

$$\text{SA} = 2\pi r^2 + 2\pi rh$$
$$= 2\pi r^2 + \frac{2\pi r.375}{\pi r^2} \ \checkmark\checkmark$$
$$= 2\pi r^2 + \frac{750}{r} \qquad (2)$$

9.1.2

$$S = 750r^{-1} + 2\pi r^2 \ \checkmark$$
$$\frac{ds}{dr} = -750r^{-2} + 4\pi r \ \checkmark$$

By min $\frac{ds}{dr} = 0$

$$\frac{-750}{r^2} + 4\pi r = 0 \ \checkmark$$
$$-750 + 4\pi r^3 = 0$$
$$r^3 = \frac{750}{4\pi} = \frac{375}{2\pi} \ \checkmark$$
$$r = \sqrt[3]{\frac{375}{2\pi}} \text{ cm} \ \checkmark \quad \textbf{OF} \quad 5\sqrt[3]{\frac{3}{2\pi}} \qquad (5)$$

9.2 9.2.1

$$f(x) = (x+2)(x-3)^2 \ \checkmark$$
$$= (x+2)(x^2 - 6x + 9) \ \checkmark$$
$$= x^3 - 6x^2 + 9x + 2x^2 + 2x + 18$$
$$= x^3 - 4x^2 - 3x + 18$$
$$p = -4 \ \checkmark \quad q = 18 \ \checkmark$$

**OF** deur vervanging

$$0 = (-2)^3 + p(-2)^2 - 3(-2) + q \ \checkmark$$

en

$$0 = (3)^3 + p(3)^2 - 3(3) + q \ \checkmark$$

en los op vir $p$ en $q$ $\checkmark \ \checkmark$ (4)

9.2.2

$$m\tan = f'(x) = 3x^2 - 8x - 3 \ \checkmark$$

By A, $3x^2 - 8x - 3 = 8 \ \checkmark$

$$3x^2 - 8x - 11 = 0$$
$$(3x - 11)(x + 1) = 0 \ \checkmark$$
$$x = \frac{11}{3} \text{ of } x_A = -1 \ \checkmark$$

$$y_A = (-1+2)(-1-3)^2$$
$$= 16 \ \checkmark$$

**OF** vervang in vergelyking

$$A(-1;16) \qquad (5)$$

IEB Copyright © 2020 

9.2.3 $f(x) = x^3 - 4x^2 - 3x + 18$

$f'(x) = 0$ ✓ (draaipunt)

$\therefore 3x^2 - 8x - 3 = 0$

$(3x + 1)(x - 3) = 0$ ✓

$\therefore x = -\frac{1}{3}$ ✓ of $x = 3$

Draaipunte $\left(-\frac{1}{3}; \frac{500}{27}\right)$ ✓ ; (3 ; 0) ✓ (5)

9.2.4 By K, $m\tan = f'(0) = -3$ ✓

$\therefore m\perp = \frac{1}{3}$ ✓

K is $(0;18)$

$\therefore h(x) = \frac{1}{3}x + 18$ ✓ (3)

9.3 9.3.1 $A = -t^2 + 5t + 8$

$A = -(0)^2 + 5(0) + 8$ ✓

$= 8$ cm$^2$ ✓ (2)

9.3.2 $\frac{dA}{dt} = -2t + 5$ ✓

by $t = 2$

$= -2(2) + 5$ ✓

$= 1$ cm$^2$ ✓ (3)

9.3.3 $\frac{dA}{dt} = 0$ ✓

$-2t + 5 = 0$

$t = \frac{5}{2}$ sekondes ✓ (2)

**[33]**

IEB Copyright © 2020

## VRAAG 10

10.1 (a) $\int 0dx = c$ ✓ (1)

(b) $\int dx = x + c$ ✓ (1)

10.2 Bepaal: $\int\left(3x^2 + \frac{1}{x}\right)dx$

$= \frac{3x}{3} + \ln x + c$

$= x^3$ ✓ $\ln x$ ✓ $+ c$ ✓ (3)

10.3 Oppervlakte $= \int_0^2 x^3 dx$ ✓

$= \left[\frac{x^4}{4}\right]_0^2$ ✓ ✓

$= \frac{16}{4} - 0$ ✓

$= 4$ ✓

en deur simmetrie $\int_{-2}^0 x^3 dx = 4$ ✓

$\therefore$ Totale oppervlakte $= 8$ eenhede$^2$ ✓

**OF**

$\int_{-2}^0 x^3 dx = \left[\frac{x^4}{0}\right]_{-2}^0$ ✓

$= -4$

$\therefore$ 8 eenhede$^2$ ✓ (7)

**[12]**

**Totaal: 150 punte**

IEB Copyright © 2020