NASIONALE SENIOR SERTIFIKAAT-EKSAMEN
NOVEMBER 2024

WISKUNDE: VRAESTEL II

NASIENRIGLYNE

Tyd: 3 uur 150 punte

Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en hulpeksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.

Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.

IEB Copyright © 2024

AFDELING A

VRAAG 1

(a)	$AO = OE$ (radii)	rede
	$\hat{E}_1 = 40°$ ($\angle$ teenoor = radii)	rede
	$\hat{D} = 140°$ (teenoorst. $\angle$ koordevierhoek)	rede
	$\hat{F}_1 = 90°$ (lyn van middelpunt halveer koord)	rede
	$\hat{A}_1 = 50°$ (raaklyn $\perp$ radius)	rede
	$\hat{B} = 50°$ (raaklyn-koord-stelling)	rede
	$A\hat{O}E = 100°$ ($\angle$ by middelpunt) / (binne$\angle\Delta$)	
(b)	Konstruksie: radii OA en OC	
	$O\hat{A}C = 90° - \hat{A}_1$ (raaklyn $\perp$ radius)	
	$O\hat{C}A = 90° - \hat{A}_1$ ($\angle$ teenoor = radii)	
	$A\hat{O}C = 2\hat{A}_1$ (binne$\angle\Delta$)	
	$\hat{B} = \hat{A}_1$ ($\angle$ by middelpunt)	

IEB Copyright © 2024

VRAAG 2

(a)	$\left(3;-\frac{9}{2}\right)$	y x
(b)	$AM = \sqrt{(-3-3)^2 + (-2-2)^2}$ $AM = 2\sqrt{13}$	Vervang orrekte formule antwoord
(c)	$m_{MA} = \frac{-2-2}{-3-3}$ $\tan\theta = \frac{2}{3}$ $\theta = 33,69°$	m_{MA} $\tan\theta = \frac{2}{3}$ $\theta = 33,69°$
(d)	$y - \frac{9}{2} = \frac{2}{3}(x+3)$ $\therefore y = \frac{2}{3}x + \frac{13}{2}$	vervang punt gradiënt antwoord in korrekte vorm
(e)	$PM = 6,5$ eenhede $MR = 6,5$ eenhede $\therefore PM = MR$ $\therefore PARM$ is 'n ruit (aangrensende sye =) **Alternatief:** $m_{PR} = \frac{\frac{9}{2} + \frac{9}{2}}{-3-3}$ $m_{PR} = -\frac{3}{2}$ $\therefore m_{MA} \times m_{PR} = -1$ $\therefore PARM$ is 'n ruit (hoeklyne halveer teen 90°)	PM MR Korrekte metode gebruik om 'n ruit te bewys

IEB Copyright © 2024

VRAAG 3

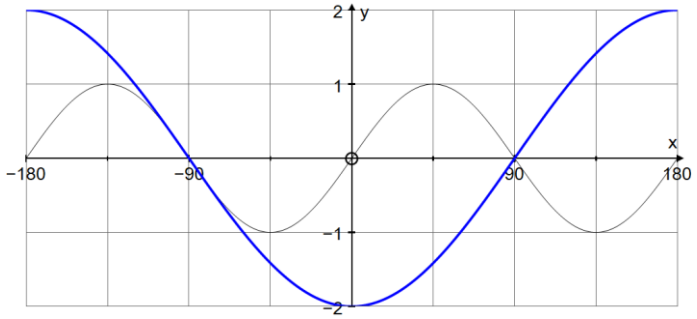
(a)	$x^2 - 2x + 1 + y^2 + 2y + 1 = 3 + 1 + 1$ $(x-1)^2 + (y+1)^2 = 5$ $\therefore \text{N}(1;-1)$ en $r = \sqrt{5}$ eenhede	Voltooi die vierkant N r
(b)	$x^2 - 2x + y^2 + 2y - 3 = 0$ vervang $y = 0$ $x^2 - 2x + (0)^2 + 2(0) - 3 = 0$ $x^2 - 2x - 3 = 0$ $x = 3$ of $x = -1$ $\therefore \text{K}(-1;0)$	$y = 0$ kwadratiese vergelyking oplossings vir x $K(-1;0)$
(c)	$m_{KL} = \frac{-1-0}{1+1}$ $m_{KL} = -\frac{1}{2}$ $\therefore m_{KJ} = 2$ $y = 2(x+1)$ $\therefore y = 2x + 2$	m_{KL} m_{KJ} $y = 2x + 2$
(d)	$\tan\theta = 2$ $\theta = 63{,}4°$ $\therefore \hat{\text{L}} = 63{,}4°$ (raaklyn-koord-stelling)	$\tan\theta = 2$ θ $\hat{\text{L}}$ en rede

VRAAG 4

(a)	$\hat{M} = 90° \left(\angle \text{ in halfsirkel}\right)$	M rede
(b)	In ΔLMN en ΔLPM (1) $\hat{L} = \hat{L}(\text{gemeenskaplik})$ (2) $\hat{M} = \hat{P}_1 (\text{beide} = 90°)$ (3) $\hat{M}_1 = \hat{N}(\text{3de } \angle)$ $\therefore \Delta LMN \|\|\| \Delta LPM(HHH)$	$\hat{L} = \hat{L}(\text{gemeenskaplik})$ $\hat{M} = \hat{P}_1$ *AAA*
(c)	$\frac{LM}{LP} = \frac{MN}{PM} = \frac{LN}{LM}$ $LM^2 = LP \times LN$ $4^2 = LP \times 8$ $LP = 2$ **Alternatief:** $4^2 = LP \times 8 (\perp \text{ uit regte } \angle)$ $LP = 2$	$\frac{LM}{LP} = \frac{MN}{PM} = \frac{LN}{LM}$ $4^2 = LP \times 8$ $LP = 2$ $4^2 = LP \times 8$ $\perp$uit regte $\angle$'e hoekpunt LP

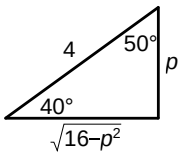
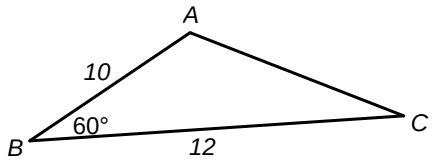
IEB Copyright © 2024

VRAAG 5

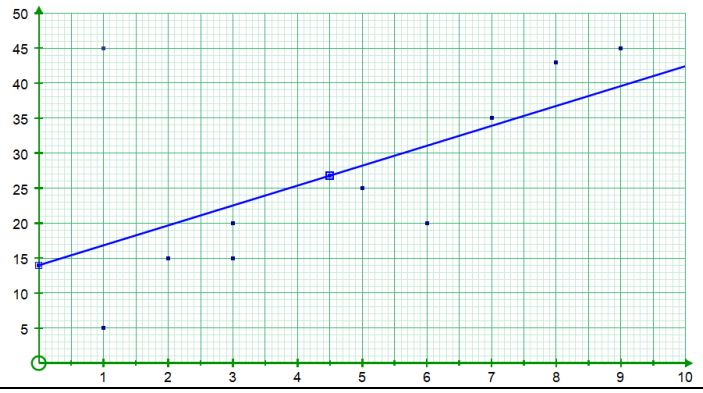
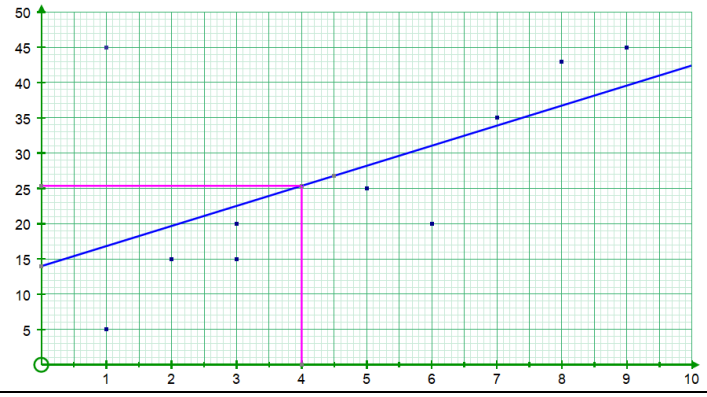
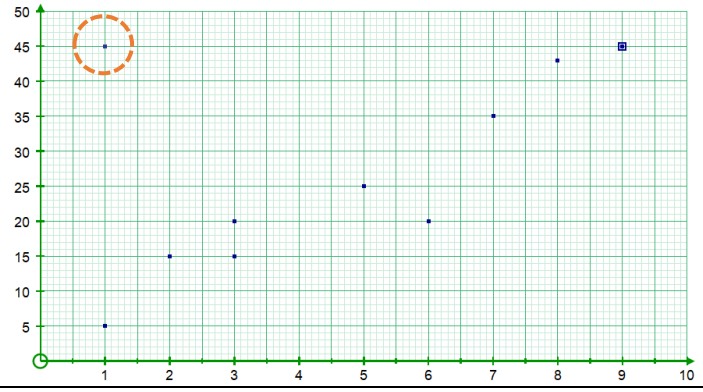
(a)	$b = 2$	b
(b)	$180°$	antwoord
(c)		cos-vorm konstante amp Korrekte x- en y-snitte
(d)	2	antwoord
(e)	$\theta = 90° + k180°$	$90°$ $k180°$
(f)(1)	$x \in [-90°;90°]$	$-90°;90°$
(f)(2)	$x \in (0°;90°) \cup (90°;180°)$	$(0°;90°)$ $(90°;180°)$
(f)(3)	$\therefore x \in (0°;180°)$	Interval hake
(g)	$y = -2\cos(x - 45°) + 2$	$-45°$ $+2$

IEB Copyright © 2024

VRAAG 6

(a)(1)	$= \sin(50° - 10°)$ $= \sin 40°$ $= \frac{p}{4}$	$(50° - 10°) / \sin 40°$ antwoord
(a)(2)	$= \cos^2(20°)$ $= \cos 40°$ $= \frac{\sqrt{16 - p^2}}{4}$ 	Aangrensend in diagram $\cos 40°$ antwoord
(a)(3)	$= \frac{\sqrt{16 - p^2}}{p}$	antwoord
(b)	 $AC^2 = 10^2 + 12^2 - 2(10)(12)\cos 60°$ $AC = 2\sqrt{31}$ eenhede	Diagram Cos-reël antwoord

VRAAG 7

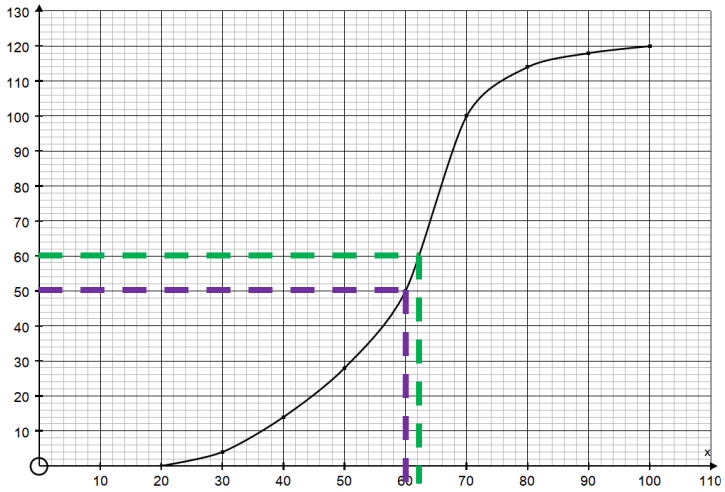
(a)	$y = 14 + 2{,}8x$	A B
(b)	$(\bar{x};\bar{y}) = (4{,}5\,;\,26{,}8)$ 	$\bar{x}$ $\bar{y}$ $y-$int lyn deur $(\bar{x};\bar{y})$ en $y-$int
(c)	R25 000 	Dui aan op grafiek Antwoord in duisende rande
(d)	$r = 0{,}6$	antwoord
(e)		Omkring op diagram
(f)	r sal toeneem	antwoord

IEB Copyright © 2024

AFDELING B

VRAAG 8

(a)



Oorspronklike Q_2 – nuwe Q_2 = 62 – 60 = 2

Mediaan posisies by 60 en 50
62
60

(b)

data is skeef na links

$\therefore \bar{x} < Q_2$

links

$\bar{x} < Q_2$

(c)

$4 - \bar{x} = -3$

$\therefore \bar{x} = 7$

$\dfrac{4 + 8 + p}{3} = 7$

$\therefore p = 9$

x	$x - \bar{x}$	$(x - \bar{x})^2$
4	–3	9
8	1	1
p	2	4

$\sigma = \sqrt{\dfrac{9 + 1 + 4}{3}}$

$\therefore \sigma = 2{,}16$

$\bar{x} = 7$

$p = 9$

Voltooi die tabel

$(p - \bar{x})^2 = 4$

$\sigma = 2{,}2$

IEB Copyright © 2024

VRAAG 9

(a)	$D\hat{F}E = 90°\left(\angle \text{in } \frac{1}{2} \text{ sirkel}\right)$ $\therefore D\hat{F}E = D\hat{E}G\,(\text{beide} = 90°)$ $\therefore EG \,\|\| \, FC(\text{omgekeerde ooreenkomstige} \angle\text{'e})$	$D\hat{F}E = 90°$ $\angle \text{in } \frac{1}{2} \text{ sirkel}$ $D\hat{F}E = D\hat{E}G$
(b)	In ΔABC en ΔDFC (1) $\hat{C}_1 = \hat{D}$ (raaklyn-koord-stelling) (2) $\hat{C}_2 = \hat{A}$ (raaklyn-koord-stelling) (3) $AC = CD$ (gegee) $\therefore \Delta ABC \equiv \Delta CFD(HHS)$ $\therefore AB = CF = 10$ eenhede **Alternatief**: In ΔABC en ΔDFC (1) $\hat{B} = 90°$ ($\angle$ in halfsirkel) $\therefore \hat{B} = \hat{F}$ (beide $= 90°$) (2) $\hat{C}_2 = \hat{A}$ (raaklyn-koord-stelling) (3) $AC = CD$ (gegee) $\therefore \Delta ABC \equiv \Delta CFD(AAS)$ $\therefore AB = CF = 10$ eenhede	$\hat{C}_1 = \hat{D}$ raaklyn-koord-stelling $\hat{C}_2 = \hat{A}$ raaklyn-koord-stelling $AC = CD$ HHS
(b)	$3(10) = 5DE$ $DE = 6$ $FD^2 = 26^2 - 10^2$ (Pythagoras) $FD = 24$ $\therefore FE = 24 - 6 = 18$	$DE = 6$ $FD = 24$
(d)	$\frac{CG}{26} = \frac{18}{24}$ (eweredigheidstelling EG||FC) $\therefore CG = 19{,}5$ eenhede	$\frac{CG}{26} = \frac{18}{24}$ eweredigheidstelling EG||FC $CG = 19{,}5$ eenhede

IEB Copyright © 2024

VRAAG 10

(a)	Sye eweredig	rede
(b)	$\hat{D}_1 = \hat{A}(\|\|\|)$ $\therefore$ DF 'n raaklyn aan die sirkel is by D (omgekeerde raaklyn-koord-st.)	$\hat{D}_1 = \hat{A}$ omgekeerde raaklyn-koord-st.
(c)	$\hat{C} = \hat{F}(\|\|\|)$ $\therefore$ GCFD is koordevierh. $(\text{omgekeerde } \angle\text{e selfde seg})$	$\hat{C} = \hat{F}$ omgekeerde $\angle$e selfde seg

VRAAG 11

(a)	$C\hat{A}B = 90° - \theta(\text{binne} \angle \Delta)$ $\frac{AC}{\sin(180° - \theta)} = \frac{10}{\sin(90° - \theta)}$ $AC = \frac{10\sin\theta}{\cos\theta}$ $AC = 10\tan\theta$	$90° - \theta$ $\frac{AC}{\sin(180° - \theta)}$ $\frac{10}{\sin(90° - \theta)}$ $\frac{10\sin\theta}{\cos\theta}$
(b)	$\tan\theta = \frac{DC}{AC}$ $\tan\theta = \frac{15}{10\tan\theta}$ $\tan^2\theta = \frac{3}{2}$ $\tan\theta = \pm 1{,}22\ldots$ $\theta = 50{,}77°$	tan-verhouding sub van AC $\tan^2\theta = \frac{3}{2}$ $\tan\theta = \pm 1{,}22\ldots$ $\theta = 50{,}8°$

IEB Copyright © 2024

VRAAG 12

(a)	LK: $= \frac{\cos 2x + \cos x}{\sin 2x - \sin x}$ $= \frac{2\cos^2 x + \cos x - 1}{2\sin x \cos x - \sin x}$ $= \frac{(2\cos x - 1)(\cos x + 1)}{\sin x(2\cos x - 1)}$ $= \frac{\cos x + 1}{\sin x}$ $\therefore$ LK = RK	$\cos 2x$ $+\cos x$ $-\sin x$ $2\cos^2 x - 1$ $2\sin x \cos x$ $(2\cos x - 1)(\cos x + 1)$ $\sin x(2\cos x - 1)$
(b)	$\frac{\cos 2x + 1}{\sin 2x} = 1$ $\frac{2\cos^2 x - 1 + 1}{2\sin x \cos x} = 1$ $\frac{2\cos^2 x}{2\sin x \cos x} = 1$ $\frac{\cos x}{\sin x} = 1$ $\tan x = 1$ $x = 45° + k180° \quad k \in \mathbb{Z}$ **Alternatief:** $\frac{\cos 2x + 1}{\sin 2x} = 1$ $2\cos^2 x - 1 + 1 = 2\sin x \cos x$ $2\cos^2 x - 2\sin x \cos x = 0$ $2\cos x(\cos x - \sin x) = 0$ $\cos x = 0$ or $\cos x - \sin x = 0$ $x \neq \pm 90° + k360°$ $\cos x = \sin x$ $\tan x = 1$ $x = 45° + k180°$ $k \in \mathbb{Z}$	$2\cos^2 x - 1$ $2\sin x \cos x$ $2\cos^2 x$ $\frac{\cos x}{\sin x}$ $\tan x = 1$ $x = 45° + k180°$ $2\cos^2 x - 1$ or $1 - 2\sin^2 x$ or $\cos^2 x - \sin^2 x$ $2\sin x \cos x$ $\cos x - \sin x = 0$ $x \neq \pm 90° + k360°$ $\tan x = 1$ $x = 45° + k180°$

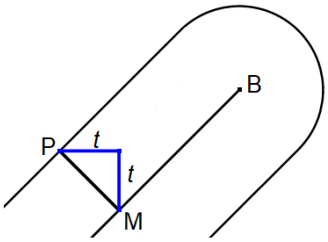
IEB Copyright © 2024

VRAAG 13

(a)	$\mathrm{E}(9;10)$ $\therefore b = 10$ $a = 9 - 25$ $a = -16$ $\therefore \mathrm{C}(-16;10)$	$\mathrm{E}(9;10)$ $b = 10$ $a = -16$
(b)	$\hat{\mathrm{D}} = 90° \left(\tan \perp \mathrm{rad}\right)$ $\mathrm{CD} = 15 (\mathrm{pythag})$ $\therefore (x+16)^2 + (y-10)^2 = 225$	$\hat{\mathrm{D}} = 90° \left(\tan \perp \mathrm{rad}\right)$ $\mathrm{CD} = 15$ $(x+16)^2 + (y-10)^2 = 225$
(c)	$r \in (0;10)$ or $r \in (40;\infty)$	$r \in (0;10)$ $r \in (40;\infty)$

IEB Copyright © 2024

VRAAG 14

(a)	$M_{AB}(8;4)$ $m_{AB} = 1$ $\therefore m_{PQ} = -1$ PQ: $y - 4 = -1(x - 8)$ $y = -x + 12$ $MP^2 = (x-8)^2 + (y-4)^2$ *sub* MP *en* $y = -x + 12$ $\left(2\sqrt{2}\right)^2 = (x-8)^2 + (-x + 12 - 4)^2$ $\left(2\sqrt{2}\right)^2 = (x-8)^2 + (-x + 8)^2$ $8 = x^2 - 16x + 64 + x^2 - 16x + 64$ $0 = 2x^2 - 32x + 120$ $x = 10$ *or* $x = 6$ $\therefore (6;6)$ en $(10;2)$ **Alternatief:** $M_{AB}(8;4)$ $m_{AB} = 1$ $\therefore m_{PQ} = -1$ $\therefore \frac{\Delta y}{\Delta x} = \frac{t}{t}$ $t^2 + t^2 = \left(2\sqrt{2}\right)^2$ $t = 2$ $(6;6)$ en $(10;2)$  	$M_{AB}(8;4)$ $m_{AB} = 1$ en $m_{PQ} = -1$ PQ: $y = -x + 12$ $MP^2 = (x-8)^2 + (y-4)^2$ $0 = 2x^2 - 32x + 120$ $(6;6)$ $(10;2)$
(b)	$V = \pi r^2 H + \frac{4}{3}\pi r^3$ $V = \pi\left(2\sqrt{2}\right)^2 \times 8\sqrt{2} + \frac{4}{3}\pi\left(2\sqrt{2}\right)^3$ $V = 379{,}13$ *eenhede*^{3}	$V = \pi r^2 H + \frac{4}{3}\pi r^3$ $H = 8\sqrt{2}$ $r = 2\sqrt{2}$ $V = 379{,}13$ *eenhede*^{3}

Totaal: 150 punte

IEB Copyright © 2024